First, let's review the key points about circuits:

Charge conservation $\Rightarrow$ current in $=$ current out
so $\sum I = 0$ into all points in a circuit.

Conservation of energy $\Rightarrow$ Voltage drop around any loop is net of zero.
$\sum V = 0$.

For both, it is important to get the signs right.
- Positive charge flows from $+$ to $-$.
- Resistors drop voltage and batteries add voltage, along the direction of $I$.

With these two rules, we can analyze the behavior of resistive circuits.

**Example:**

\[ V_B - I_1 R_1 = 0 \Rightarrow V_B = I_1 R_1 \]

or $-V_B + I_1 R_1 = 0 \Rightarrow$ same.

\[ V_B - I_2 R_2 - I_3 R_3 = 0 \Rightarrow V_B = I_2 (R_2 + R_3) \]

$\Rightarrow I_0 = \left( \frac{V_B}{R_1} \right) + \left( \frac{V_B}{R_2 + R_3} \right)$

$\Rightarrow P = I_0 V_B = V_B^2 \left[ \frac{1}{R_1} + \frac{1}{R_2 + R_3} \right]$
Example:

What is \( V_{AB} \), the voltage difference between points A and B?

It is zero because there is no resistance; \( V_{\text{drop}} = IR \)

What is \( I \)? \( I = \frac{10V}{(5+5) \Omega} = 1A \).

What if I break the circuit?

\( V_{AB} = ? = 10V \)

\( I \rightarrow 0 \) so no voltage dropped in the 5-\( \Omega \) resistors.

How about this?

\( V_{AB} = ? = 0 \) because the short circuit shunts all the current. In both cases, no voltage drop.

It is important to remember that a real battery has some non-zero \( r_{\text{int}} \) so \( I \neq \infty \).

What is \( V_{\text{drop}} \) across 5-\( \Omega \)'s?

\( = 0 \)
A few more examples before moving on...

\[ 1V \quad 2\Omega \quad 4\Omega \quad 6\Omega \]

In what part of the circuit is the current largest?

→ In the battery.

What is the total power dissipated by the circuit?

\[ P = IV, \text{ but finding } I \text{ could take some arithmetic.} \]

Instead

\[ P = \frac{V^2}{R} \quad \text{for each resistor.} \]

\[ \therefore P_{\text{total}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6}{12} + \frac{3}{12} + \frac{2}{12} = \frac{11}{12} \text{ W} \]

What if it was a 10V battery? \( V^2 \rightarrow 100 \text{ times as big.} \)

Which bulb is brightest in the following circuit?

\[ 10V \quad C \]

\[ \therefore \text{All 3 bulbs are identical.} \]

\[ 10V \quad B \]

\[ \text{Wall the horizontal bulb gives no light.} \]

\[ \sum V = 0 \text{ around outer loop } \Rightarrow \text{10V drop across } A \text{ and across } B. \]

\[ \therefore \text{C has no voltage drop. } \Rightarrow \text{no current.} \]
Which of the following circuits dissipates more power?

\[ \text{IV} \quad \text{or} \quad \text{IV} \]

Well, \( P = IV \) so we just need to find largest \( I \), or \( P = \frac{V^2}{R_{\text{tot}}} \) \( \Rightarrow \) find smallest \( R \).

Plugging into arithmetic is not necessary... instead think conceptually.

- On the left, \( R_{\text{tot}} \) is 2\( \Omega \) plus something less than 1\( \Omega \).
- On the right, \( R_{\text{tot}} \) is 1\( \Omega \) plus something less than 1\( \Omega \).

\[ \Rightarrow R_{\text{tot}} \text{ is less on the right} \Rightarrow \text{Power greater on the right.} \]

Think about what happens as I add more resistors in parallel --- the upper \( R_{\text{eff}} \) only decreases.

This is useful in analyzing a circuit: Find what part dominates and/or what part is independent. (Good circuit design consists of independent blocks).
Measuring voltage and current.

Current can be measured using a current meter or ammeter. Old fashioned "analog" meters use a needle that is deflected to some reading. They work as follows:

![Diagram of a galvanometer with a coil of wire, a permanent magnet, and a needle.]

Current in the coil causes the coil to be torqued by the magnet (details next quarter). Torque is proportional to $I$. So a needle attached to the coil rotates. A torsional spring balances it.

$\therefore I \rightarrow$ displacement of the needle.

This is the idea of a "galvanometer."

I don't consider the details important here -- next quarter -- but how it can be used in a circuit is important as an example.

To measure $I$ in this circuit, we put an ammeter into the circuit.

If the meter has any resistance, it will change $I_0$. BAD.

So, it should have some low $R_c$ = resistance of the coil.
Typically, \( R_e \approx 10^{-2} \) because it is many winds of very thin wire. That is fine if \( R \gg 10^{-2} \).

A galvanometer has some specified maximum current that "pegs" the needle. How could you measure larger currents?

I could also do: And so on, to reduce the fraction of current in (A). But the 1Ω may dissipate high \( I^2R \).

We can use this to measure voltage too. E.g. What is \( V \) across a resistor.

Now, we want the voltmeter to draw no current, well at least very little current compared to the current through \( R_1 \). \( \Rightarrow \) \( V \) has large \( R \).

\( \Rightarrow \) \( V \) corresponds to 1mA. You just have to scale the needle's deflection.
To see if this makes sense, suppose we have a circuit like this.

\[\text{Diagram of circuit with a voltmeter and resistors.}\]

Where \(\nabla\) has an internal resistance of \(1 \text{M} \Omega\).

If \(\nabla\) reads 1 Volt, what is the battery's voltage?

The \(1 \text{M} \Omega\) in the voltmeter is not large compared to the circuit's resistance, so

\[\text{Diagram of circuit with a meter and resistors.}\]

\[1 \text{ V} \Rightarrow 1 \text{mA in both branches (since equal resistance).}\]

\[2 \text{mA here}\]

\[V = 1 \text{mA} \times 1 \text{M} \Omega + 2 \text{mA} \times 1 \text{M} \Omega\]

\[= 1 + 2 = 3 \text{ V.}\]

Without the meter, there is really 1.5 V across \(R\).
Now, we want to make things a little more interesting by adding capacitors.

Consider what happens in this circuit.

\[ V_b \quad \quad R \quad \quad c \]

Analyzing this is trickier. The voltage across the capacitor depends on how much charge is stored on it. Recall \( q = CV \) or \( V = q/c \).

And the two plates in a capacitor don't touch so current doesn't flow across it, instead charge just builds up on it.

Such a circuit has changing behavior.

Let's explicitly determine what happens by first imagining that the battery is not connected.

\[ \text{The switch is open and there is no charge on the capacitor.} \]
When the switch is closed, what happens?

By Kirchhoff’s rule, the voltage around the loop must sum to zero, so

\[ V_b - VR - \frac{q}{C} = 0 \]

\[
\begin{array}{c}
\text{battery} \\
\text{resistor} \\
\text{capacitor}
\end{array}
\]

At first, there is no charge on the capacitor so

\[ V_b = IR \quad \Rightarrow \quad I = \frac{V_b}{R} \]

as if the capacitor is at zero volts (it is!).

But with current flowing charge gets put onto the capacitor (electrons pulled from one side \(\frac{1}{2}\) stuck onto the other). So \(q\) increases.

\[ \Rightarrow \quad V_b - IR - \frac{q}{C} = 0 \]

or

\[ \frac{V_b}{R} - I - \frac{q}{RC} = 0 \]

\[ \Rightarrow \quad I = \frac{V_b}{R} - \frac{q}{RC} = I_0 - \frac{q}{RC} \]

\(\uparrow\) initial current

\(\Rightarrow\) current decreases as charge stored.

\(\Rightarrow I\) is a function of time.
\[ I(t) = \frac{V_b}{R} - \frac{q}{RC} \]

\[ I(t) = \frac{dq}{dt} = \text{rate at which charge is pulled from one side of } C \text{ and stuck on the other.} \]

\[ \frac{dq}{dt} = \frac{V_b}{R} - \frac{q}{RC} \]

\[ \Rightarrow \quad \frac{dq}{V_b - \frac{q}{R}} = dt \]

\[ \frac{dq}{CV_b - q} = \frac{1}{RC} dt \]

Integrate

\[ \frac{1}{RC} \int_0^t dt' = \int_0^q \frac{dq'}{CV_b - q'} = -\int_0^q \frac{dB'}{q' - CV_b} \]

\[ \frac{t}{RC} = -\ln \left( q - CV_b \right) + \ln \left( -CV_b \right) \]

\[ = -\ln \frac{q - CV_b}{-CV_b} \]

\[ \frac{q - CV_b}{-CV_b} = e^{-t/RC} \Rightarrow \quad q - CV_b = -CV_b e^{-t/RC} \]

\[ q = CV_b \left[ 1 - e^{-t/RC} \right] \]
Check:  At t=0  \( q = 0 \).
At \( t \to \infty \)  \( q \to CV_b \) the most it can pump.

What is the voltage across the capacitor as a function of time?

\[
V_c = \frac{q(t)}{C} = V_b \left[ 1 - e^{-t/RC} \right]
\]

Note the limits again.
What is the current through a function of time?

\[
I(t) = \frac{V_b - V_c(t)}{R} = \frac{V_b}{R} - \frac{V_b}{R} \left[ 1 - e^{-t/RC} \right]
\]

At \( t=0 \), \( I(0) = \frac{V_b}{R} \) as we saw above.

So:

[Graphs]

Comment on:
\[
\frac{dV_c}{dt} = \frac{I}{C} \quad \Rightarrow \quad V \propto I \quad \text{impedance}
\]
is complex.

[Could also have taken derivative of \( q(t) \)]
The RC in the exponential is called the
time constant. After RC seconds,
$e^{-t/RC}$ changes by $1/e$.

For example, 1kΩ and 1μF $\Rightarrow$ RC = 1ms.
$\Rightarrow$ too fast to see by eye
but also very slow compared to computer speeds $\sim$1GHz.
$\Rightarrow \approx$ 1ns.

If $R=1kΩ$, what must C be to have RC < 1ns?

$10^{-9} = 10^3 C \Rightarrow C \leq 1 \mu F$.

Or, can change R to make it small.
$\Rightarrow$ speed of CPU's increase as feature size shrinks.

**DEMO charging on an oscilloscope.**
Discharging a capacitor.

If we remove the battery, what happens?

Nothing. Once charged, the capacitor stays charged, since there is no path for the current to flow back.

If we short across where the battery was, then current can flow.

\[ V_c = IR = \frac{q}{C} \]

\[ \Rightarrow I = \frac{q}{RC} \]

As before,

\[ I(t) = \frac{dq}{dt} \]

But now the charge flows the other direction and \( V_c \) decreases.

\[ \frac{dq}{dt} = -\frac{q}{RC} \quad \Rightarrow \quad \frac{dq}{q} = -\frac{1}{RC} \, dt \]

\[ \Rightarrow \quad \ln \frac{q}{q_0} = -\frac{t}{RC} \quad \Rightarrow \quad q = q_0 e^{-t/RC} \]

\[ \Rightarrow \quad V_c = V_0 e^{-t/RC} \quad \Rightarrow \quad I(t) = \frac{d}{dt} \left( \frac{q_0}{RC} e^{-t/RC} \right) = I_0 e^{-t/RC} \]
We can put this charge-discharge to use to make a light blink.

\[ V_b = 100 \text{ V} \]

Lamp = neon light, which breaks down at nearly 100 V.

So, C charges until \( V_C > \) neon breakdown. Then a flash as current flows through the (now) low R of the lamp. Then recharge & repeat.

Changing R changes the charging time constant.

Demo 64.60
Finally, I'd like to spend a couple minutes talking about issues in real circuits.

First, ground. All that mattered in our Kirchoff loops was the voltage difference. But if we connect two circuits together then, they should be referenced to each other somehow.

E.g. if you connect two car batteries (charging with jumper cables) it must be + to + and - to -.

To simplify this, it is useful to define a "ground." Any connections between two circuits can have the same ground.

It is called ground (or "earth" in "english") because it is often actually connected to the ground so you have the same ground as your neighbor.

It is the zero voltage reference we can all agree upon.

Symbol is $\frac{1}{\Xi}$ or $\nabla$
E.g. in modern 14 μm chips, use 2.5V. But I want 1.25V for analog purposes.

That is why your home's power should have grounded plugs... and for safety.

**DEMO fuse.**

Fuse on "hot" side not ground.

Box around the circuit should be grounded so any accidental short goes to gnd not you.