Assignment 8: Power in Resistive Electric Circuits

**Part A**

Using Ohm's law:

\[ I = \frac{V}{R_{tot}} \]

\[ R_{tot} = \text{Total Resistance} = R + R_{int} \]

Since \( R \) and \( R_{int} \) are in series,

\[ V = E \] which is the voltage supplied by the battery.

Thus \[ I = \frac{E}{R+R_{int}} \]

**Part B**

Voltsmeter will measure the potential difference between points 1 and 2.

Using Ohm's law:

\[ V = IR = \frac{ER}{R + R_{int}} \] \hspace{1cm} \text{Using result for } V \text{ about.}

**Part C**

Power dissipation in a given \( R \): \( P_R = \frac{SV^2}{R} \]

**Part D**

\[ P_R = \frac{SV^2}{R} = \frac{E^2R}{R + R_{int}} \]

**Part E**

\[ P_R = \frac{SV^2}{R} = \frac{E^2R}{(R + R_{int})^2} \] \hspace{1cm} \text{Using result from Part A}

**Part F**

Power dissipation in the whole circuit is the sum of the power dissipated in each element of the circuit:

\[ P_{\text{Battery}} = \frac{E^2 R_{int}}{(R + R_{int})^2} \]

Power dissipated between points 1 and 2 is 0 since there is no circuit element (other than the wire) between pts. 1 and 2.

\[ P_{\text{Total}} = P_R + P_{\text{Battery}} = \frac{E^2 R_{int} + E^2}{(R + R_{int})^2} \]

**Part G**

\[ P_{\text{Total}} = \sqrt{E} \] using result from Part A.
**Part A**

Calculate \( V_{ab} = V_a - V_b \)

Since ammeter is ideal it has \( R = 0 \)

So the voltage drop from \( a \) to \( b \) is caused by the resistor \( R \) so using ohm's law

\( V_{ab} = \frac{E}{R} \).

Since \( R_{total} \) for the circuit is \( R + R_v \) and EMF supplied by battery is \( E \) we can calculate \( \frac{E}{R} = \frac{E}{R + R_v} \)

\( V_{ab} = \frac{E}{R + R_v} = V_{meter} \)

Alternatively we can calculate \( V_{ab} \) by tracing the circuit in the other direction to \( E \).

\( V_a + \frac{E}{R} - E = V_b \) \( \Rightarrow V_{ab} = E - \frac{E}{R} \)

Combining \( 0 \) a \( \Rightarrow E - \frac{E}{R} - V_{ab} = 0 \)

\( V_{meter} = V_{ab} = E - \frac{E}{R} = E \left( 1 - \frac{R}{R+R_v} \right) \)

As \( R_v \) gets much larger than \( R \) the Voltmeter's Measurement Approaches \( E \) very closely.

**Part B**

Given \( E = 7.50 \text{V} \), \( R = 0.45 \Omega \)

Since we want \( V_{meter} \) to be within \( 1.0\% \), we want \( V_{meter} = 0.99 E \)

So using results from part A we must solve for \( R \) in the equation

\( 0.99 E = E \left( 1 - \frac{R}{R+R_v} \right) \)

\( \Rightarrow (0.99 \times 7.50 \text{V}) = \left( 1 - \frac{0.45 \Omega}{R} \right) \times 7.50 \text{V} \)

\( \Rightarrow R = 44.6 \Omega \)
Resistance of a Heater

Part A

1500 W heater & 120V outlet

Since a heater can be viewed as a resistor, and the outlet can be viewed as a battery that supplies 120V, we can use

\[ P = IV \]

To solve for \( I = \frac{P}{V} \)

\[ 1500 \text{ W} = I \cdot (120 \text{ V}) \Rightarrow I = 12.5 \text{ A} \]

**Classic Units**

\[
\begin{align*}
W &= 300 \text{ J/sec} \\
V &= 3 \text{ V} \text{ Dc} \\
W/V &= 5/3 = 1.666 \text{ A}
\end{align*}
\]

**Part B**

\[ V = IR \]

Ohm's Law \( \Rightarrow R = \frac{V}{I} = \frac{120 \text{ V}}{12.5 \text{ A}} = 9.6 \Omega \)

**Part C**

First calculate the mass of air in the room:

\[ m = \left( \text{Volume of Room} \times \text{Density of Air} \right) \]

\[ m = (30 \times 5 \times 8) \text{ m}^3 \times (1.29 \text{ kg/m}^3) = 144 \text{ kg} \]

**Heat Capacity**

Energy required to heat an object is:

\[ Q = m \cdot c \cdot \Delta T \]

Energy required to heat an object is:

\[ Q = (144 \text{ kg}) \times 1006 \text{ J/kg} \times (10 \text{ C}) = 1.45 \times 10^6 \text{ J} \]

So, \( Q = (144 \text{ kg}) \times 1006 \text{ J/kg} \times (10 \text{ C}) = 1.45 \times 10^6 \text{ J} \)

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expansion 1 K (1 K = 1000 J), \( Q = 1450 \text{ KJ} \)

Since \( W = \frac{Q}{c} \text{ sec} \), to find time to heat room we use:

\[ \frac{Q}{T} = \frac{1450 \text{ KJ}}{(1200 \text{ W})} = \frac{(1450 \text{ KJ})}{(2000 \text{ sec})} = 1.21 \text{ min} \]
**Part A**

\[ d = 1.205 \text{ cm} \]
\[ \rho = 2.75 \times 10^{-8} \text{ u}\Omega \text{ m} \]
\[ \text{Emp} = 1.0084, t^2, -0.00141T + 0.0004 \% \quad T \text{ in sec.} \]

\[ R = \rho \frac{L}{A} = \text{Resistance of a wire with length } L, \text{ cross-sectional area } A \text{ and resistivity } \rho. \]

\[ A = \left(\frac{1}{2}\right)\pi d^2, \quad V = \text{Emp} \cdot L \quad \text{since the field is in the wire and points along the length} \]

\[ I = \frac{V}{R} = \frac{\text{Emp} \cdot L}{\rho \frac{L}{A}} = 1.24A \quad \text{[Make sure to convert 1205 cm to m. Since } \rho \text{ is in units of u}\Omega \text{ m]} \]

**Part B**

**B.1**

\[ \dot{y} = \frac{dy}{dt} = \text{rate at which charge passes through a cross-section of the wire. Integrate both sides to find } \]

Total charge \( Q \) that flows through a cross-section from \( t_1 \) to \( t_f \) is

\[ Q = \int_{t_1}^{t_f} \dot{y} \, dt \]

**B.2**

\[ \int (kt^2 - bt + c) \, dt = \frac{1}{2} \, at^3 - \frac{1}{2} \, bt^2 + ct \]

From Part A:

\[ \text{Emp} = \frac{\rho\pi}{A} \]

\[ \int_0^b \lim_{t \to 12.1} \, dt = 2.1 \text{ C} \]
Problem 2E.69

Part A
Total Resistance, \( R_{tot} = 24 \Omega \)
\[ V_{tot} = 8V - 4V = 4V \]
\[ I_{tot} = \frac{V_{tot}}{R_{tot}} = \frac{1}{6} A \]

Going from a to d (watching the signs of the voltage drops)
\[ V_a = 0 \text{V} \quad V_b = 1 \text{V} \quad V_c = 4.0 \text{V} = 0.5 \text{V} \quad V_d = V_4 \]
\[ \Rightarrow V_{ab} = V_a - V_b = -1 \text{V} \]
\[ V_{bc} = V_b - V_c = -3.5 \text{V} \]
\[ V_{cd} = V_c - V_d = 3.5 \text{V} \]

Part B
Terminal Voltage includes internal resistance
\[ V_{tot} = V_{int} + (\frac{1}{2})(\frac{1}{6}) V \]
Terminal Voltage = \( V_{tot} + 4V = 4.08 \text{V} \)

Part C
Motor resistance \( R_{mot} \)
\[ I_{mot} = \frac{5 - 4 - 10.3}{24.5 \Omega} = -0.26 \text{A} \]
\[ V_{terminal} = (\frac{1}{2})I_{mot} + 4V = 3.87 \text{V} \]

Problem 25.76
\[ \frac{V}{\sqrt{R}} \quad R = \frac{9L}{A} \Rightarrow \left(\frac{\text{Motor Internal} + \text{Paralleled Lamps}}{\frac{1}{2}}\right) \]
\[ P = \frac{V^2}{R} \text{ but since } \frac{V}{\sqrt{R}} \text{ then } = \frac{\text{Speed}}{\text{Time}} \]
\[ \Rightarrow \text{Energy} = \text{Power} \times \text{Time} \]
\[ \Rightarrow \text{Energy} = \frac{3}{2}RT \text{ using } R \text{ from above} \]

Kuhneman's Loop Rule

Voltage drops across all resistors in a loop must equal the voltage supplied by the battery in the loop.

A) \quad \begin{array}{c}
A \\
B \\
A \\
\end{array}

B) \quad \begin{array}{c}
A \\
B \\
A \\
\end{array}

C) \quad \begin{array}{c}
A \\
B \\
A \\
\end{array}
Throw the Switch

Part A

\[ \Delta V_c = 0 \] when the switch is closed

Since C and the switch form a loop with no battery

\[ \Delta V_b = 0 \] when the switch is closed since IT and C form a loop with no voltage source.

\[ \Delta V_a = E \] when the switch is closed since voltage drop across it must equal the voltage supplied by the battery.

\[ \Delta V_a = \frac{2E}{3} \] when the switch is closed B, C are bypassed (short circuit)

So B, C are correct.

Part B

B.1) When switch is closed C is short-circuited so its voltage drop = 0.

This leaves B, D in parallel. The bulbs have resistance R

The \[ R_{eq} = R + \left( \frac{1}{R} + \frac{1}{R} \right)^{-1} = R + \frac{R}{2} = \frac{3R}{2} \]

B.2) \[ V = 5R \] \[ V = \frac{E}{R_{eq}} = \frac{2E}{3R} \] so \[ \Delta V_a = \frac{E}{R} = \frac{3E}{2} \]

For switch open:

\[ R_{eq} = R + \left( \frac{1}{R} + \frac{1}{2R} \right)^{-1} = \frac{5R}{3} \]

\[ V = \frac{E}{R_{eq}} = \frac{3E}{5R} \Rightarrow \Delta V_a = \frac{3E}{5} \] So statement A is correct

A

From loop rule: A, D, E bulbs must have voltage drop equal to E.

Since E is fixed and we know A changes when the switch is closed, D must change as well. Therefore statement D is also false.

\[ \Delta V_b = \frac{E}{3} \] when switch is closed. Open switch removes C from loop so \[ \Delta V_b = \frac{E}{3} \] so statement C is false.

Open switch on B, C, D. From a loop, \[ \Delta V_b + \Delta V_c = \Delta V_b \] but \[ \Delta V_b = \Delta V_c \Rightarrow \Delta V_b = \frac{2E}{3} \]

\[ R_{eq} = \frac{5R}{3} \Rightarrow \Delta V_a = \frac{2E}{3} \] \[ \Delta V_b = \frac{E}{3} \] \[ \Rightarrow \] statement B is false.

Only statement A is correct.
Equivalent Resistors

**Part A**

\[ R_{eq} = \left( \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} + R_3 + R_4 = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_4 \]

- **Parallel:**
  \[ \frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \]
  - All in **Series**

**Part B**

Since switch is open, R_7 has no current.

\[ R_{12C} = R_1 \frac{V}{R_2} \Rightarrow \left( \frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} \]

\[ R_{12C} = \frac{R_2 (R_1 + R_2)}{R_1 + R_2 + R_6} \]

\[ R_{Total} = R_{12C} + R_3 + R_4 + R_5 \]

\[ = \frac{R_2 (R_1 + R_2)}{R_1 + R_2 + R_6} + R_3 + R_4 + R_5 \]

**If switch is closed:**

\[ R_{eq} = \frac{R_7 R_6}{R_7 + R_6} \]

\[ R_{Total} = R_{12C} + R_3 + R_7H + R_5 \]

\[ = \frac{R_2 (R_1 + R_2)}{R_1 + R_2 + R_6} + R_3 + R_5 + \frac{R_7 R_6}{R_7 + R_6} \]
Using Kirchhoff’s Loop rule @ pt 1
1. \( \varepsilon = \varepsilon_a - \varepsilon_b \)

**Summary Voltage Drops about Small Loop**
2. \( \varepsilon = 7\varepsilon_a + 2\varepsilon_b \)

**Summary Voltage Drops about Outer loop**
3. \( 24 - 2\varepsilon - 7\varepsilon_a = 0 \)

Combining 1 + 3:
\[
24 - 10\varepsilon_a = 3\varepsilon_b
\]
\[
\Rightarrow \varepsilon_b = \frac{1}{3}(24 - 10\varepsilon_a) \quad \text{(7)}
\]

Substitute 4 into 3:
\[
\varepsilon = 7\varepsilon_a + \frac{2}{3}(24 - 10\varepsilon_a)
\]
Problem 26.86

A) \[ Y = RC \]
\[ R_{tot} = \frac{C}{RC} \]
\[ Y = \frac{1}{3 \times 10^{-5}} \]

R) When the switch is closed, the capacitor will begin charging:
\[ Q(t) = Q_{max}(1 - e^{-\frac{t}{RC}}) \]
\[ V(t) = \frac{Q(t)}{C} \]
\[ V_{C(t)} = V_{max}(1 - e^{-\frac{t}{RC}}) \]

To find \( V_{max} \), we consider the circuit after it has charged for a long time. Then there will be no current flow, so \( AV_B = 0 \) in the resistor. Thus, the circuit will look like:
\[ 12 = V_2 = V_1 \]
\[ V_1 = \frac{Q}{C} \Rightarrow V_2 = 2V_1 \]

So we get 3V across the 3 \( \mu \)F capacitor.

Thus:
\[ V_C(t) = 8V \left( 1 - e^{-\frac{t}{RC}} \right) = 8V \left( 1 - e^{-\frac{t}{3 \times 10^{-5}}} \right) \]

Problem 26.91
\[ Y = \frac{V_B}{I} = \frac{Q}{RC} \Rightarrow Q = RCY = 3.12 \times 10^{-6} \, C \]

Problem 26.42
A) \[ Q = VC = 1.65 \times 10^{-4} \, C \]
B) \[ Q(t) = Q_{max}(1 - e^{-\frac{t}{RC}}) \]
\[ ln \left( 1 - \frac{Q(t)}{Q_{max}} \right) = \frac{-3 \times 10^{-3}S}{RC} \]
\[ \Rightarrow R = \left( \frac{3 \times 10^{-3}S}{Q_{max}} \right) \left[ ln \left( 1 - \frac{1.05 \times 10^{-4}C}{1.65 \times 10^{-4}C} \right) \right]^{-1} \approx 463 \, \Omega \]
Part C)

We have \( Q_{\text{in}} = 0.99 \), \( Q_{\text{max}} \). Solve,

\[
Q_{\text{max}} = Q_{\text{in}} \left(1 - e^{-\frac{T}{\tau}}\right) \quad \text{for } T
\]

\[
0.99 = \left(1 - e^{-\frac{T}{\tau}}\right)
\]

\[
\tau = RC = (463 \mu \text{s} \times 5.5 \mu \text{F}) = 2.53 \times 10^{-3} \text{ s}
\]

\[
T = -\tau \ln(0.01) = 0.0126 \text{s}
\]

Problem 26.36

\( V_0 = 15 \text{V} \) Voltage becomes

Capacitor begins to

Discharge

\( V(t) = V_0 e^{-\frac{t}{\tau}} \)

\[\ln\left(\frac{V(t)}{V_0}\right) = \frac{-t}{\tau}\]

\[
\Rightarrow \quad C = \frac{R}{R \ln\left(\frac{V_0}{V(t)}}\right)} = 0.96 \mu \text{F}
\]