Homework #6

1. Introduction to Capacitance

Part A.

What property of objects is best measured by their capacitance? Ability to store charge.

\[ B \]

\[ E = \frac{q}{A \varepsilon_0} \]

Part C

\[ V = Ed = \frac{q d}{A \varepsilon_0} \]

Part D

\[ C = \frac{Q}{V} = \varepsilon_0 \frac{A}{d} \]

Part E

For an air-filled charged capacitor, decreasing the spacing between the plates will increase the capacitance. \( \therefore C = \varepsilon_0 \frac{A}{d} \)

Part F

To quadruple the capacitance, halve the plate separation, double the plate area.
2. Energy of a capacitor

**Part A**

When dielectric material was inserted, charge Q was constant.

So \( U = \frac{1}{2} QV = \frac{Q^2}{2C} \), \( C \propto k \Rightarrow U \propto \frac{1}{k} \)

Energy of the capacitor has decreased.

This is because of mutual attraction between dielectric material and capacitor, and this energy is dissipated in the resistor.

So \( \frac{U}{k} \)

**Part B**

When the battery was connected, V was constant.

So \( U = \frac{1}{2} QV = \frac{1}{2} CV^2 \), \( C \propto k \Rightarrow U \propto kU \)

This energy increase comes from the battery. The battery does work on the capacitor by moving more electrons from one plate to another.

**Prob 24.34**

**Part A.** E field between the sphere \( = \frac{kQ}{r^2} \).

\[ \Delta V = \int_{r_1}^{r_2} E \cdot dr = kQ \left[ \frac{1}{r_2} - \frac{1}{r_1} \right] = \frac{3.87 V}{r_2} \]

\( r_2 = 11.50 \text{ cm} \)

\( r_1 = 10 \text{ cm} \)
4. Problem 24.54

**Part A**

\[ \Delta C = \varepsilon_0 A \left( \frac{1}{d} - \frac{1}{d_0} \right) = 0.240 \, \text{pF} \]

\[ = 0.240 \times 10^{-12} \, \text{F} \]

\[ \frac{1}{d} = \frac{1}{d_0} + \frac{\Delta C}{\varepsilon_0 A} = 2026 \]

\[ d = 0.000493 \, \text{m} \]

\[ |d - d_0| = 1.6 \times 10^{-4} \, \text{m} \]

5. Problem 24.38

**Part A**

\[ Q = CV = CE \Delta d = (6.30 \, \text{pF}) \cdot (3 \times 10^9 \, \text{V/m}) \cdot (2 \, \text{mm}) \]

\[ = 3.78 \times 10^{-10} \, \text{C} \]

**Part B**

\[ Q = CV \cdot C \alpha K \Rightarrow Q' = 2.60 \cdot Q \]

\[ = 2.60 \times 3.78 \times 10^{-10} \, \text{C} \]

\[ = 9.83 \times 10^{-10} \, \text{C} \]
6. Prob 24.66

**Part A**

\[ \frac{1}{2} \epsilon_0 \frac{Q}{\rho} \]

Now total capacitance is from

\[ Q = C \Delta V, \quad \Delta V \Rightarrow \text{total voltage drop} \]

\[ \Delta V = \Delta V_1 + \Delta V_2 \]

and \[ Q = C_1 \Delta V_1 = C_2 \Delta V_2 \]

\[ C_1 = C_2 = \frac{\epsilon_0 A}{d-a} \]

\[ \Rightarrow \Delta V = \Delta V_1 + \Delta V_2 = \Theta \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \]

\[ \Rightarrow C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1}{2} = \frac{\epsilon_0 A}{d-a} \]

**Part B**

In terms of \[ C_0 = \frac{\epsilon_0 A}{d} \]

\[ C = C_0 \cdot \frac{d}{d-a} \]
7. Prob 24.60

**Part A**

\[ V_{AB} = 210 \text{ V} \]

\[ V_i = \text{voltage drop at } d \text{ from } A \]

\[ = \frac{1}{C_1} V_{AB} \quad (\because \quad Q = CV, \quad \frac{Q}{C_1} = V_i, \quad \frac{Q}{C_2} = V_2, \quad V_{AB} = V_i + V_2) \]

\[ = Q \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \]

\[ \Rightarrow V_i = \frac{Q}{C_1} = \frac{1}{C_1 + \frac{1}{C_2}} V_{AB} \]

Similarly

\[ V_2 = \text{voltage drop at } C \text{ from } A \]

\[ = \frac{1}{C_2} V_{AB} \quad \frac{1}{C_1 + \frac{1}{C_2}} \]

\[ \Rightarrow V_{CD} = |V_i - V_2| = \left| \frac{\frac{1}{C_1} - \frac{1}{C_2}}{\frac{1}{C_1} + \frac{1}{C_2}} \right| V_{AB} \]

\[ C_1 = 3 \text{ mF}, \quad C_2 = 6 \text{ mF} \]

\[ = \frac{\frac{1}{3} - \frac{1}{6}}{\frac{1}{3} + \frac{1}{6}} \cdot 210 = 70 \text{ V} \]

**Part B**

If switch is closed, it is equivalent to

\[ \Rightarrow V_{AB} = \frac{1}{2} \cdot 210 = 105 \text{ V} \]

\[ C_{eff} = C_1 + C_2 \]

\( \therefore \) \( C_{eff} = C_1 + C_2 \)
Part C

\[ V_{db} = \frac{1}{2} V_{AB} = 105 \text{ V} \]

Part D

\[ V_{ac} = V_{ad} = \frac{1}{2} V_{AB} = 105 \text{ V} \]

("C & d are at the same voltage")

Part E

\[ V_{cb} = V_{db} = 105 \text{ V} \]

Part F

\[ \text{initially} \quad Q_1' = Q_2' = 0 \]

\[ \text{net change in charge} \]

\[ \Delta Q = Q_2 - Q_1' = (6.00 \mu \text{F} - 3.00 \mu \text{F}) \cdot 105 \text{ V} \]

\[ = 3.15 \times 10^{-4} \text{ C} \]

2. Prob 24.30

Part A

\[ U = \frac{1}{2} CV^2 = \frac{Q^2}{2C} \]

\[ C = \varepsilon_0 \frac{A}{d} \]

\[ \Rightarrow U \propto d \quad \Rightarrow \quad U = U \cdot \frac{d'}{d} = 8.24 \cdot \frac{1.45}{2.50} = 4.80 \text{ V} \]

Part B

\[ U = \frac{1}{2} CV^2 \]

("potential source disconnected")
\[ C \propto \frac{1}{d} \]
\[ \frac{1}{d} \Rightarrow \quad U' = U \cdot \frac{d}{d'} = 8.28 \cdot \frac{2.50}{1.45} = 14.3 \, \text{J} \]

\[ \frac{1}{2} \varepsilon_0 A \frac{1}{V} \Rightarrow \quad Q = \frac{2U}{V} = 1.25 \times 10^{-9} \, \text{C} \quad \left( \frac{U}{V} = 2.5 \times 10^{-9} \right) \]

\[ B = \left( \text{unknown} \right) \]
\[ \text{in a cylinder} \]
\[ E = \frac{\lambda}{2\pi \varepsilon_0 l} \quad \Delta V = \frac{\lambda}{2\pi \varepsilon_0} \frac{l}{r} \left( \frac{r_2}{r_1} \right) \]
\[ = \frac{\Theta}{2\pi \varepsilon_0} \frac{l}{r} \left( \frac{r_2}{r_1} \right) \]

\[ \Rightarrow \quad Q = C \Delta V \Rightarrow \quad C = \frac{2\pi \varepsilon_0 l}{l \left( \frac{r_2}{r_1} \right)} = \frac{Q}{\Delta V} = 3.65 \times 10^{-10} \, \text{F} \]

Since \( \varepsilon_0 = 8.85 \times 10^{-12} \, \text{F/m} \), \( l = 16.8 \, \text{m} \)

\[ \frac{r_2}{r_1} = 19.8 \]

\[ \text{Prob 24.65} \]

Part A

\[ \Rightarrow Q = C V \Rightarrow \quad k C_0 V = C_0 V \]
\[ \Rightarrow \quad k = \frac{V}{V} = \frac{48}{12.5} = 3.84 \]
Part B

\[ C_1 = \frac{kC_0}{2}, \quad C_2 = \frac{2}{3}C_0 \Rightarrow \text{parallel connection} \]

\[ C = C_1 + C_2 \]

\[ = \frac{1}{3}(k+2)C_0 \]

Now \[ Q = CV = C_0V_0 \]

\[ \Rightarrow V = \frac{3}{k+2}V_0 = 24.7 \, V \]

11. Prob 24.57

\[ \frac{C_1}{C_2} \]

\[ C_1 = 3.90 \, \mu F \]

\[ C_2 = 5.60 \, \mu F \]

\[ 660 \, V \]

Part A

\[ Q_1 = C_1V = 2.57 \times 10^{-3} \, C \]

Part B

\[ Q_2 = C_2V = 3.70 \times 10^{-3} \, C \]

Part C

\[ V_1 = 660 \, V \]

Part D

\[ V_2 = 660 \, V \]
Part E

\[ Q = |Q_1 - Q_2| = 1.72 \times 10^{-3} \text{ C} \]

The total charge will be redistributed to each capacitor since they are parallel-connected.

\[ Q = Q_1 + Q_2 = C_1 V + C_2 V \]

\[ Q_1 = \frac{C_1}{C_1 + C_2} Q = 2.57 \times 10^{-3} \text{ C} \]

\[ Q_2 = \frac{C_2}{C_1 + C_2} Q = 3.70 \times 10^{-3} \text{ C} \]

Part F

\[ V_1 = V_2 = \frac{Q_1}{C_1} = 118 \text{ V} \]

Part H

\[ V_1 = V_2 = 118 \text{ V} \]

12. Capacitors W/ Partial Dielectrics

\[ C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{\varepsilon_0 A}{d} \cdot \frac{2k}{k+1} \]

The horizontal \( \frac{1}{C_1} \) is equivalent to the capacitors which are connected in series.
vertical capacitor is equivalent to the capacitor which are connected in parallel,

\[
\begin{align*}
C &= C_1 + C_2 \\
&= \frac{\varepsilon_0 A}{d} f_k + \frac{\varepsilon_0 A}{d} (1-f) \\
&= \frac{\varepsilon_0 A}{d} [f_k + 1-f] \quad (2)
\end{align*}
\]

from 1 & 2

\[f_k + 1 - f = \frac{2k}{k+1} \Rightarrow f = \frac{1}{k+1}\]

13. Capacitor W/ Partial Dielectric

we have considered the same problem in Prob 12.

\[
C = C_1 + C_2 = \frac{\varepsilon_0 A}{d} [f_k + 1-f] \]

\[C_0 = \frac{\varepsilon_0 A}{d} \Rightarrow C = C_0 [f_k + 1-f] \]