Midterm Review

So far, we have covered relativity and quantum mechanics.

Today, I'll review them, try to point out the key concepts that you should understand, and do some sample problems.

Relativity is about transformations between coordinate systems moving relative to each other.

A coordinate system is a means of measuring positions. As we saw in physics 1, it doesn't matter what coordinate system you use, the laws of physics still apply. So, we get to choose one that makes the problem easier (both mathematically and conceptually).

So, I measure elevation from the floor, because it is easier than from sea level or the center of the earth. We can always transform to another coordinate system if needed.

\[ y = y' + D \]

Just an offset.
If I’m moving in a car, it is still convenient to measure things in a coordinate system stuck to the car, but now the offset is time dependent, \[ D = vt. \]

\[ \text{call it } u \text{ not } v, \text{ see below.} \]

If someone in the back seat throws something toward the front of the car at a speed \( v \), the an observer on the ground sees it moving at a speed \( v + u \) in their frame.

This is the old "Gallilean" view.

Einstein’s relativity adds a few things that change the picture, based simply on the postulate that every observer should measure the same speed for light.

Since "speed" involves not just position, also time we found that both position and time get changed as we transform between coordinate systems, also known as reference frames.
The need to include time is clear when we think about events. An event occurs at a specific space and time.

Different reference frames can have different time coordinates just like they have different spatial coordinates. The offsets are obvious e.g., in eastern time vs. pacific time.

So, an event is \((x, y, z, t)\) in one frame and \((x', y', z', t')\) in another frame.

By considering propagation of a light pulse as viewed by observers in two reference frames, one moving relative to the other, we derived the requirement that the different observers must have different views of "clock speed" from each other → time dilation.

Similarly, we got length contraction.

More generally, we have the Lorentz transformations.
Lorentz transformations:

\[ x' = \gamma (x - ut) \quad y' = y \quad z' = z \]

\[ t' = \gamma (t - \frac{ux}{c^2}) \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}} \]

iff \((x = 0, t = 0)\) when \((x' = 0, t' = 0)\), and the primed system moves toward \(\hat{x}\) relative to the unprimed system.

\[ S \rightarrow S' \]

\[ \uparrow \quad \rightarrow u \]

This means that an event measured with the \(S\) frame at \((x, t)\) corresponds to \((x', t')\) which is an event at \(x' \neq x\) and \(t' \neq t\) as measured in the \(S'\) frame.

What if we know the coordinates in the \(S'\) frame i.e., \((x', t')\) and want to find the corresponding coordinates in the \(S\) frame? Just swap \(x \leftrightarrow x'\) and \(t \leftrightarrow t'\) and \(u \rightarrow -u\).

\[ x = \gamma (x' + ut') \quad t = \gamma (t' + \frac{ux'}{c^2}) \]
A sample problem:

Observer A flies past observer B at a speed of $0.5c$. They each reset their clocks at the moment they pass. When observer A measures 10 seconds on his clock, he turns on a light in his hand and simultaneously another light 5m in front of him. From B’s reference frame, when & where are the lights lit?

Call B the $S'$ frame, because we know A is moving past him at $v = +0.5c$.

So we know the coordinates for the two events (each light being lit) in the $S'$ frame $\left(x'_1, t'_1\right)$ and $\left(x'_2, t'_2\right)$.

Observer B measures these as $\left(x_1, t_1\right)$ and $\left(x_2, t_2\right)$.

\[
\begin{align*}
x_1 &= \gamma \left( x'_1 + ut'_1 \right) = \gamma \left( 0 + (0.5c)(10) \right) = 5\gamma c \\
t_1 &= \gamma \left( t'_1 + ux'_1/c^2 \right) = \gamma \left( 10 + 0.5c \right) (0/c^2) = 10\gamma \\
x_2 &= \gamma \left( x'_2 + ut'_2 \right) = \gamma \left( 5 + (0.5c)(10) \right) = 5\gamma \left( 1+c \right) \\
t_2 &= \gamma \left( t'_2 + ux'_2/c^2 \right) = \gamma \left( 10 + (0.5c)(5)/c^2 \right) = \gamma \left( 10 + 0.5c \right)
\end{align*}
\]

$t_2 \neq t_1$, even though $t'_2 = t'_1$. Simultaneity is relative...
We then derived the velocity transformations based on

\[ v = \frac{dx}{dt} \quad \text{and} \quad v' = \frac{dx'}{dt'}, \]

where the \( dt' \neq dt \) is important.

We get

\[ v' = \frac{v - u}{1 - u'v'c^2} \quad \text{and} \quad v = \frac{v' + u}{1 + u'v'c^2} \]

Similarly, if we know \( v' \) and \( u \) we find \( v \).

Then we derived the momentum and energy definitions:

\[ \vec{P} = \gamma m \vec{v} \]

\[ K = (\gamma - 1) mc^2 \]

\[ E = K + mc^2 = \gamma mc^2 \]

\[ E = \sqrt{p^2c^2 + (mc^2)^2} \]
We also used the relativistic Doppler shift:

\[ f = f_0 \sqrt{\frac{c+u}{c-u}} \]

just like the old fashioned Doppler shift, but also works for light.
We saw from the photoelectric effect that light is quantized into packets with $E=hf$.

Review photoelectric effect.

$\Rightarrow$ Photons = "particles" of light.

This then explains the observed atomic spectra:
- Specific atoms emit light of only specific frequencies $\Rightarrow$ specific energies.
- A whole set of these energies.

These energy levels were explained, at least for hydrogen, by the Bohr model.

$$V = -\frac{e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \frac{1}{2} \mathbf{p}^2 = m v r \equiv \frac{nh}{2\pi}$$

$$c = \frac{\varepsilon_0 n^2 \hbar^2}{\pi ma^2}$$

$$V = \frac{e^2}{2\varepsilon_0 nh}$$

$$E = U + k = -\frac{m e^4}{8\varepsilon_0 n^2 \hbar^2}$$

$\Rightarrow$ Explained measured spectra
We then talked about lasers as an example of these energy levels.

Wave ↔ Particle duality. ⇒ deBroglie's

\[ p = \frac{h}{\lambda} \quad \text{or} \quad \lambda = \frac{h}{p}. \]

Particles are waves and have all the usual wave properties like diffraction.

We also get uncertainty principle: since you cannot simultaneously specify the position of a wave and its wavelength, or its frequency at any time.

All we can really say is the wave's function. But that gives us a prediction for the probability of finding it somewhere.

\[ P(x) = |\psi(x)|^2 = \psi^*(x) \psi(x) \]

Then we talked about finding \( \psi \) with Schrödinger's equation... but that should all be fresh.