Magnetic Field.

Just as an electric field allows us to calculate the force on a charged particle, a magnetic field allows us to calculate the force on a moving charged particle.

And, $\vec{B}$ fields are produced by moving charges.

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{8\vec{v} \times \hat{r}}{r^2} \perp \vec{v} \text{ and } \hat{r}.$$  

What is $\vec{B}$ at various points around a moving charge?

If we have a wire carrying a current, then it is "many moving charges."

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, dl \times \hat{r}}{r^2}$$

And the complete $\vec{B}$ can be obtained by integrating.
Ampere's law allowed simpler determination of $\vec{B}$ in cases where symmetry can be used.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

This makes some problems easy, but you must know how to use it.

E.g. A long straight wire. $\vec{B}$ at a distance $r$?

Choose a path that uses the symmetry and simplifies the integral.

$\vec{B}$ points tangent to a circle, so $\vec{B} \cdot d\vec{l} = B dl$

$B$ constant around circle by symmetry.

$$\oint \vec{B} \cdot d\vec{l} = B 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}.$$
How about for a loop?

Ampere's law doesn't help because there is no exploitable symmetry.

So we must explicitly integrate each $d\mathbf{B}$.

You should be able to do that, AND to know what the $\mathbf{B}$ field lines will look like for various geometries. E.g.

This acts like a magnetic dipole, like a bar magnet.

Where the field from a permanent magnet comes from a net ~ from microscopic effects.
We can determine the forces caused by a magnetic field with

\[ \vec{F} = q \vec{v} \times \vec{B} \quad \text{or} \quad \vec{F} = I \vec{L} \times \vec{B} \]

Again, we may have to integrate over \( d\vec{L} \) if it is not uniform.

Note that \( \vec{F} \neq \vec{v} \) and \( \vec{B} \).

We did various problems with this such as the force on a wire carrying current.

We also spent a lot of time on the force and torque on a loop.

What is force and torque if \( \vec{B} \) points into the page? Out? Left? Right? At an angle out of the page?

Found that \( \vec{\tau} = \vec{\mu} \times \vec{B} \) where \( \vec{\mu} = I \vec{A} \).
We calculate magnetic flux just as we did for electric flux:

\[ \Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \]

is flux through a 2D area.

Flux through a closed 3D surface is

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

since no magnetic charge, only dipoles.

But flux was useful for induction.

\[ E = -\frac{d\Phi_B}{dt} \]


Lenz's law is that the sign is such that it tries to oppose the changing flux.

If the EMF can cause a current to flow, then one does, but the EMF is there even if a current cannot flow.
A good example problem is to calculate the induced current in a wire of resistance $R$, near a current carrying wire:

\[
I(t) = 2t^2 + 1
\]

\[\Phi_B = \int \vec{B} \cdot d\vec{A} = \int \frac{\mu_0 I(t)}{2\pi x} \, dA = \int_0^h \int_0^{L+w} \frac{\mu_0 I(t)}{2\pi x} \, dx \, dy
\]

\[= \frac{\mu_0 I(t)}{2\pi} \ln \frac{L+w}{L} = \frac{\mu_0 h}{2\pi} \ln \frac{L+w}{L} \left(2t^2 + 1\right)
\]

\[\Rightarrow \mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 h}{2\pi} \ln \frac{L+w}{L} \left(4t\right)
\]

\[I_{\text{induced}} = \frac{\mathcal{E}}{R} = -\frac{2\mu_0 h t}{\pi R} \ln \frac{L+w}{L}
\]

Which direction?
We saw that since $E$ is essentially an electric field, 
$E = \vec{E} \cdot d\vec{r}$.

changing $\vec{B}$ induces $\vec{E}$.

AND

changing $\vec{E}$ induces $\vec{B}$.

This came through the "displacement current".

$\oint \vec{B} \cdot d\vec{r} = \mu_0 I$

becomes generalized to

$\oint \vec{B} \cdot d\vec{r} = \mu_0 \left( I_{\text{conductor}} + \varepsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{enclosed}}$
Since a changing \( E \) field produces a \( B \) and vice versa, oscillating fields propagate as waves.

We used Maxwell's laws to show that EM waves propagate at a speed of

\[
V = \frac{1}{\sqrt{\varepsilon \mu}} \quad \text{where in vacuum, } \varepsilon = \varepsilon_0 \text{ and } \mu = \mu_0 \Rightarrow V = c.
\]

You should understand how that comes about.

EM waves carry energy, the Poynting vector quantifies that as the power/area,

\[
\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad \text{points \perp to both } \vec{E} \text{ and } \vec{B}.
\]

So the intensity is \(|\vec{S}|\) actually \( I = \frac{|\vec{S}|}{\sec}\).

EM waves also carry momentum, radiation pressure

\[
P_{\text{rad}} = \frac{I}{c}.
\]

Be able to describe EM waves as a sinusoidally varying wave function.
Then we discussed the nature of light.

Reflection & Refraction

\[ \theta_i = \theta_f \]
\[ n \sin \theta_i = n_2 \sin \theta_f \quad \text{where} \quad n = \frac{c}{V}, \quad \lambda = \frac{\lambda_0}{n}. \]

Total internal reflection:
\[ \sin \theta_{\text{crit}} = \frac{n_2}{n_1} \]

Dispersion: \( n \) is a function of \( \lambda \). \( \Rightarrow \) rainbows and prisms.

Polarization & polarizing filters. Polarization by reflect. Little math, but make sure you understand the concepts.

Optics: Just apply reflection & refraction.

Good examples are finding the image location for a mirror (plane, concave, convex) or a refracting object (plane, concave, convex).

You should be able to do that either with ray drawings or \[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]

or \[ \frac{n_1^2}{s} + \frac{n_2^2}{s'} = \frac{n_2^2-n_1^2}{R} \]

Thin lenses are two refracting surfaces close together so we can treat their surfaces as one. Then
\[ \frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Again, you should be able to analyze image formation from a lens.
Interference and diffraction

The most important point here is that light is a wave so it interferes:

constructively if $\Delta \phi = m \pi \quad (m=0,1,2,\ldots)$
destructively if $\Delta \phi = m \pi \quad (m=1,3,5,\ldots)$

There are many ways that we can arrange such a phase shift:

- reflection from a higher $n$
- change in path length ($Lx$)
- change in wave length ($k$)
- delay in time

The various configurations lead to very different equations

$$d \sin \theta = m \pi \quad \Rightarrow \text{constructive}$$

or

$$a \sin \theta = m \pi \quad \Rightarrow \text{destructive}$$

Memorizing these is like memorizing a table of integrals. It is better to understand how to integrate.
Exam details.

It will be similar in format to the midterm, but longer and harder. On some of the questions you may be asked to show your work. So, be detailed in your explanations there to get credit.

Many of the conceptual questions are designed to be easy if you understand the concepts. If you are not sure --- stop and review the concepts for yourself then reconsider the question.

You may bring a handwritten note sheet. You may bring a calculator.

How to study:

- Redo the homework, make sure you can do it yourself.
- Reread the chapters (and examples) to refresh.
- Explain the material to friends.
- Discuss the conceptual questions.
- Take the practice final (ready by Wednesday) to check if you understand.
Faraday's Law and Induced EMF

Consider a rectangular loop of wire with sides x and y placed in a region where a uniform magnetic field exists. The resistance of the loop is R. Initially, the field is perpendicular to the plane of the loop and is directed out of the page. The loop can rotate about either the vertical or horizontal axis, passing through the midpoints of the opposite sides, as shown.

a). Find the flux through the loop.
b). If the magnetic field steadily decreases from B to zero during a time interval \( T \), what is the magnitude of the induced emf?
c). In which direction would current flow in the loop?

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a). The flux through the loop is \( \Phi_B = \int \overrightarrow{B} \cdot d\overrightarrow{A} \) integrated over the area of the loop.

In this problem, \( \overrightarrow{B} \) is uniform, that is, it does not vary across the loop, so the integral is easy. We still, however, have to account for the vectors. \( \overrightarrow{B} \) points out of the page, which I'll call \( \hat{z} \). The area, \( d\overrightarrow{A} \), either points into the page (-\( \hat{z} \)) if I were to call a clockwise current positive, or out of the page (\( \hat{z} \)), if I were to call a counterclockwise current positive. I'll choose the "counterclockwise is positive" coordinate system. So \( d\overrightarrow{A} = dA \hat{z} \).
So, the integral is simple.

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} = \int \vec{B} \hat{z} \cdot d\vec{A} \hat{z} = \int B dA \hat{z} \cdot \hat{z} \]
\[ = \int B dA = B \int dA = BA = Bxy \]

(To make sure you understand this, think about how it would change if \( \vec{B} \) were a function of position instead of uniform, or if \( \vec{B} \) pointed along \( \hat{x} \) rather than \( \hat{z} \).)

b). If the magnitude of the magnetic field were to decrease steadily to zero in a time interval \( T \), we could describe the field as a function of time as,

\[ \vec{B}(t) = B_0 (1 - t/T) \]

though we need to keep in mind that this is only valid for \( 0 \leq t \leq T \).

Since \( \vec{B} \) is changing with time, the flux is changing, so by Faraday's law,

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \left[ B(t) \times \hat{y} \right] = -xy \frac{dB}{dt} \]
\[ = -xy \frac{d}{dt} \left( B_0 (1 - \frac{t}{T}) \right) = xy \frac{B_0}{T} \]
c). We can find the direction of the current either algebraically from $E$, or by Lenz's law. Algebraically, $E = IR$ from Ohm's law, so

$$I = \frac{E}{R} = \frac{XY Bo}{RT}$$

where the fact that this is positive means that $I$ flows counter-clockwise, due to my coordinate system choice (on the first page).

Using Lenz's law should give the same answer. So, the current will flow in a direction that would produce a $B$ field that is in a direction that would oppose the change in $\Phi_B$. The change in $\Phi_B$ is that it is decreasing, that is, the $B$ field pointing out of the page is decreasing in magnitude, so the induced current will try to add more $B$ out of the page. By the right hand rule, that requires the current to be counter-clockwise.
Young and Freedman Problem 34.99

When an object is placed at the proper distance to the left of a converging lens, the image is focused on a screen which is placed a distance of $s_1$ to the right of the lens. A diverging lens is now placed a distance of $s_2$ to the right of the converging lens, and it is found that the screen must be moved a distance of $x$ farther to the right to obtain a sharp image. What is the focal length of the diverging lens?

Since the new lens is a diverging lens, we know it must go between the converging lens and the original image location to move the second image farther away. So the setup is as drawn above.

We can apply the thin lens equation to the diverging lens using the image from the converging lens (Image1) as the object for the diverging lens. So,

\[ \frac{1}{d} + \frac{1}{d+x} = \frac{1}{f} \]

where $d$ is the distance from the diverging lens to Image1.
We use \(-d\) in the equation because imagel is not on the "incoming light" side.

So,

\[
f = \frac{1}{d+x} - \frac{1}{d} = \frac{d - d - x}{d(d+x)} = \frac{-d(d+x)}{x}
\]

We don't know \(d\) directly, but we can find it from \(s_1\) and \(s_2\). \(d = s_1 - s_2\). So,

\[
f = \frac{(s_2 - s_1)(x + s_1 - s_2)}{x}
\]
A glass plate with a thickness of d, an index of refraction of n, and plane parallel faces, is held with its faces horizontal and its lower face a distance of h above a printed page. Find the position of the image of the page formed by rays making a small angle with the normal to the plate.

We use a point on the page as the object, so the lower surface of the glass slab produces an image at \( s_i \), where

\[
\frac{1}{h} + \frac{n}{s_i} = \frac{n-1}{R} = 0
\]

\[ \Rightarrow \text{for plane} \]

\[ \Rightarrow s_i = -hn \]

Since this is negative, it is an image below the glass. That image then acts as the object for the upper surface of the glass. For that, the image-object relation is:

\[
\frac{n}{d+hn} + \frac{1}{s_{1M}} = \frac{1-n}{R} = 0
\]

\[ (d+hn \text{ is positive since on incoming side}) \]

\[ \Rightarrow s_{1M} = -\frac{d+hn}{n} = -\frac{d}{n} - h \text{ from the top surface} \]

Since it is negative, it is below the top surface. Measured from the page, this is

\[ y_{1M} = h + d - h - \frac{d}{n} = d - \frac{d}{n} \]
A plane mirror is located at the origin. A converging lens with focal length 5.00 m is located at $x = 1.00$ m. An object is placed at $x = 31.0$ m.

A). What is the location of the final image, as seen by an observer looking toward the mirror through the lens?

B). Is the image in Part A real or virtual?

C). What is the magnitude of the magnification of the image found in Part A?

D). Is the image in Part A upright or inverted?

A). First, we find the location of the image produced by the lens. Then use that as the object for the mirror.

The position of the image from the lens is

$$\frac{1}{x_0} + \frac{1}{x_1} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{x_1} = \frac{1}{f} - \frac{1}{x_0} = \frac{x_0 - f}{x_0 f}$$

$$\Rightarrow \quad x_1 = \frac{x_0 f}{x_0 - f} = \frac{(31.0 - 1.0)(5.0)}{31.0 - 1.0 - 5.0} = 6.0 \text{ m}$$

Since $x_1 > 0$, the image is 6.0 m to the left of the lens, i.e., 5.0 m to the left (behind) the mirror.
This image is virtual because of the mirror, but we can still use it as an object for the mirror. The image that the mirror then produces is at $x_2$, where

$$\frac{1}{x_{01}} + \frac{1}{x_2} = \frac{1}{f}$$

for mirror.

$$\Rightarrow x_2 = -x_{01}$$

So,

$$x_2 = +5.0 \text{ m}$$

Now, this image acts as an object for the lens. Since it is located on the opposite (outgoing) side of the lens, it comes in negative in the image-object relation.

$$\frac{1}{x_2-x_L} + \frac{1}{x_3-x_L} = \frac{1}{f}$$

$$\frac{1}{x_3-x_L} = \frac{1}{f} + \frac{1}{x_2-x_L} = \frac{x_2-x_L + f}{f(x_2-x_L)}$$

$$x_3 = x_L + \frac{f(x_2-x_L)}{x_2-x_L+f} = 1.0 + \frac{5.0(5.0-1.0)}{5.0-1.0+5.0}$$

$$\boxed{x_3 = 3.22 \text{ m}}$$

is the final image location.

The key was using the image from one step as the object for the next.
B). The image is real because light rays really pass through that point. This is true even though one of the intermediate images was virtual (behind the mirror).

C). To find the total magnification, we have to use \( m = \frac{s'}{s} \) for each step combined. For the first pass through the lens, the object to lens distance is \( s = x_0 - x_L = 30 \text{ m} \). The image distance is \( s' = x_1 = 6.0 \text{ m} \). So, \( m_1 = \frac{-6}{30} = -\frac{1}{5} \).

The mirror has no magnification. The second pass through the lens gives \( m_2 = \frac{-s'}{s_2} = \frac{(x_2 - x_L)}{x_L - x_2} = \frac{2.22}{4.0} = \frac{1.11}{2.0} \).

So,

\[ m = m_1 m_2 = -\frac{1}{5} \cdot \frac{1.11}{2.0} = -0.11 \]

So its magnitude is \( 0.11 \)

D). \( m < 0 \) so the image is inverted.