Last time we looked at extensions of the two slit interference: multiple slits and a single slit.

With multiple slits, we get constructive interference when \( d \sin \theta = m \lambda \)

Obtained by assuming lines are parallel and each slit adds its wave in path length more than the one above.

The intensity is

\[
I = N^2 I_{\text{ indiv}}
\]

because \( N \) waves add together and \( I = \frac{E^2}{\mu_0 c} \) squared.

This made for a very sharp intensity pattern because conservation of energy requires that higher intensity in the peaks means narrower peaks.

We saw how this can be used to precisely measure the wavelength of a light source, or the separation or spacing for a grating.
We looked at a single slit of finite width and saw that we could determine its diffraction pattern by treating each point in the slit as a source of waves.

Here, it is easiest to calculate the location of minima rather than maxima.

The first minima occurs when two pairs of points (one from the top half of the slit and one from the same position on the bottom half) differ by phase $\pi$ due to path length.

\[
\left(\frac{a}{2} \sin \Theta \right) \frac{2\pi}{\lambda} = m \pi \quad m = 1, 3, 5, \ldots
\]

\[
\frac{a}{2} \sin \Theta = \frac{2\pi}{\lambda} m
\]

\[
a \sin \Theta = m \lambda \quad m = 1, 3, 5, \ldots
\]

All other pairs of points cancel simultaneously.
The maxima are harder to pinpoint because some pairs may interfere constructively while others destructively, e.g., if

\[ \frac{a}{2} \sin \theta = \frac{2}{8} \]

then some point pairs (every 4th) give destructive but "every 8th" pairings are constructive.

So we get broad maxima between the minima.

Central peak is broader and brighter.

The location, on a screen a distance \( R \) away, of the minima is

\[ y = R \tan \theta = R \sin \theta = R \frac{m^2}{a} \quad m = 1, 3, 5, \ldots \]

Note that as \( a \to 0 \), \( y \to \infty \). \( \Rightarrow \) A single point

\[ \Rightarrow \text{No interference.} \]
The intensity from a single slit is

\[ I = I_0 \left[ \frac{\sin \left( \frac{\pi a \sin \theta}{\lambda} \right)}{\frac{\pi a}{\lambda} \sin \theta} \right]^2 \]

[The derivation of this is not worthwhile, though I wish we had time to introduce phasors.]

What is important is that it shows how the intensity decreases as we move away from the central peak.

That is consistent with what we'd expect from a ray model, but in consistent with the equal intensity in a two-slit or diffraction grating where interference dominates.
Now, we can put together the analysis of a wide single slit and many narrow slits.

What we get from many not-so-narrow slits is a convolution of the diffraction pattern based on the distance between the slits and the width of the slits.

Demo
So far, we have only considered slits. Of course, the same interference applies with other geometries. E.g. a circle

\[ d \rightarrow \text{Airy disk} \]

Just like a slit, but messier math (we won’t do).

\[ d \sin \theta = 1.22 \lambda \]

for first minima (dark ring). So the angular width of the central bright spot is

\[ \sin \theta \approx \theta = 1.22 \frac{\lambda}{d} \]

As \( d \rightarrow 0 \) it becomes a point source

\[ \nabla \]

so if we had plane waves coming in, spherical waves go out. This causes “resolving power” issues.
Suppose that we wanted to observe a distant star. We'd use a telescope the lens of which focuses parallel rays to a point.

![Diagram of a telescope](image)

Parallel rays are plane waves.

If the telescope has a small opening, of diameter $D$, then diffraction produces an Airy disk out of the parallel rays.

In the extreme case of a point, $D \rightarrow 0$, then parallel rays (plane waves) become spherical waves, and no focus at all.

This is how telescopes can become "Diffraction limited".

A point source becomes a spot of size $\theta = 1.22 \lambda / D$. 
So two nearby stars can merge

\[
\begin{align*}
\Delta \Delta & = \Delta \Delta \\
\end{align*}
\]

looks like one, no matter how small your pixels are.

Two objects are "said to be" resolved if their peaks differ by more than 1.22 \( \lambda / D \). Rayleigh's criterion.

This is not a fundamental limit, just a guide. In reality, you might be able to distinguish them at 1.22 \( \lambda / D \) with enough signal processing. But not much closer.

Homework problem on this using your eye.
Note that all of this depends on the wavelength. What is the fundamental connection?

Interference relates to phase shifts and path length differences relate to phase shifts by \( \Delta \varphi = \frac{2\pi}{\lambda} \Delta x \) because phase is \((kx - wt)\).

So, we saw ways to use interference to measure \(\lambda\).

And most recently, resolving of \(1.22 \lambda / D\) can be improved by reducing \(\lambda\).

If you want to use photolithography for chip production, you want small \(D\) \(\Rightarrow\) need small \(\lambda\).

Or, alternatively, if you have a diffraction grating with a very small \(d\), then the maxima are at \(\Theta = m \lambda / d\). To probe small \(d\) requires small \(\lambda\), of the same order as \(d\).
To see the interference from atomic scale separations, we need correspondingly small \( \lambda \).

With small \( \lambda \) we can probe the atomic spacing in a crystal. E.g., salt, \( d \approx 0.3 \text{nm} \) \( \Rightarrow \) need \( \lambda \approx 0.1 \text{nm} \) about 1000\(^{th}\) of visible. \( \Rightarrow \) X-rays.

\[
\begin{array}{ccccccc}
\text{Na} & \text{Cl} & \text{Na} & \text{Cl} & \text{Na} & \text{Cl} & \\
\text{O} & \text{O} & \text{O} & \text{O} & \text{O} & \text{O} & \\
\end{array}
\]

\[
\begin{array}{ccccccc}
\text{Cl} & \text{B} & \text{Cl} & \text{Na} & \text{Cl} & \text{Na} & \\
\text{O} & \text{O} & \text{O} & \text{O} & \text{O} & \text{O} & \\
\end{array}
\]

\[
\begin{array}{ccccccc}
\text{Na} & \text{Cl} & \text{Na} & \text{Cl} & \text{Na} & \text{Cl} & \\
\text{O} & \text{O} & \text{O} & \text{O} & \text{O} & \text{O} & \\
\end{array}
\]

The atoms in the crystal scatter the X-rays. You can think of each atom as a tiny antenna "reflecting" the light -- absorbing and reradiating.

The "reflected" light interferes.

\[
\text{extra path length} = 2 \times d \sin \theta
\]

So, we get constructive interference when \( 2d \sin \theta = m \lambda \) \( m=0,1,2,\ldots \) Bragg condition.
The actual interference is more complicated because there are other scattering configurations such as

\[ \frac{y}{\lambda} = \frac{R_2}{a} \]

etc.

But this can be used to disentangle the repeating structure of crystals.

Note, not only does this require short \( \lambda \), so x-rays. It also requires a source of x-rays with just one \( \lambda \). "Monochromatic."

Conceptual question: Do sound waves diffract?

Yes. But \( \lambda \) is a few cm, so a slit of a few cm or less is needed. We don't normally "hear" diffraction patterns though. Why?

Many frequencies, big heads.

Conceptual question: If you bounce a laser off a CD you get a diffraction pattern. Suppose you did the same off an LP (vinyl). Would the bright spots be farther apart than for the CD?

No, closer. Since \( \sin \theta = \frac{\lambda}{d} \), \( d \) is bigger for an LP.