Last time we reviewed wave interference in general and worked through Young's double slit experiment.

1). Coherent light:
   - Same 
   - Same phase

2). Different paths to one point means that two waves arrive with different phase, so they add to give interference.

   \[ \Delta r = ds \sin \theta = m \lambda \]

   \[ y = R \tan \theta = R \sin \theta \]
   \[ = R \frac{\lambda^2}{d}. \]

3). Intensity at P is

   \[ I = \frac{I_s}{4} = I_{\text{indiv}} \left( 4 \cos^2 \left( \frac{\pi y d}{\lambda R} \right) \right) = I_o \cos^2 \frac{\pi y d}{\lambda R} \]
There is another way to obtain interference bands.

Again, we need a coherent, monochromatic source, and two paths. Here it is easy to draw with just rays instead of circular wavefronts.

When do we get destructive interference? When $d_2 - d_1 = (M + \frac{1}{2})\lambda$? Actually, no! Surprisingly. If we did such an experiment, the dark regions correspond to $\Delta d = m\lambda$, which is opposite of what we might expect. Why?

**Phase shift at reflection**: Recall that for radio waves, the varying $\vec{E}$ reaching a conductor makes charges move and produce an opposite polarity $\vec{E}$ field (so total in conductor is zero). This new $\vec{E}$ radiates the reflected wave.

Opposite polarity $\Rightarrow 180^\circ$ phase shift.
Another way to think of this is to recall reflection of waves on a string.

A wave reflected at a fixed point inverts.
Inverted = 180° phase shift.

Demo 40.63, transverse wave machine.

So, in our two paths, one via a mirror, the reflected path gets a 180° phase shift at the mirror so the two rays only arrive in phase if
\[ \Delta d = (m + \frac{1}{2}) \lambda \]

This emphasizes a key point, interference is due to phase difference. Path length difference is just one way to produce a phase difference.
Thin Films

A common source of interference is light passing through thin films of material. Let's examine a simple example.

[Diagram of light passing through a thin glass slab with air on both sides, labeled as air, glass, air. A note reads: Glass slab of thickness \( t \) that is small, \( O(t) \).]

Nearly normal incident ray. (Normality required to avoid angular variation complications.)

Path 1 is reflected at the top surface. Path 2 is refracted, reflected, then refracted.

There are other reflections & refraction out the bottom, but they don't matter for interference of the two up going rays.

How thick does the glass have to be to get constructive interference? Depends on the reflective phase shifts, these are not "mirrors", but they reflect...

So, we first need to set up the rules for phase shifts upon reflection.
Demo with wave machine.

Fast to slow region inverts the wave \( \Rightarrow \Delta \Phi = \pi \).

"fixed end is very slow" is a useful mnemonic.

Slow to fast region does not invert \( \Rightarrow \Delta \Phi = 0 \).

\[ n_1 > n_2 \quad \Rightarrow \quad \Delta \Phi = \text{no change} \]

\[ n_2 > n_1 \quad \Rightarrow \quad \Delta \Phi = 180^\circ \]

So, path 1 in our thin film example has a reflective phase shift but path 2 does not.

\[ \Rightarrow \text{Constructive interference if } \Delta t = (m+\frac{1}{2})\lambda. \]

But, yet another complication is that this \( \lambda \) is the wavelength in the glass, not the air.

\[ \lambda_{\text{film}} = \frac{\lambda_{\text{air}}}{n_{\text{glass}}}. \]

\[ \Rightarrow \Delta t = (m+\frac{1}{2})\lambda = \text{constructive} \]

\[ \frac{3}{2} \Delta t = m\lambda = \text{destructive} \]

This is only for a thin film of \( n \) surrounded by air.
What if instead of a thin glass film sandwiched by air we had a thin air film sandwiched by glass.

This is actually much easier to implement than a thin glass film.

Now, the reflective phase shift is at the 2nd reflection. So,

Constructive: \[ \Delta \phi = (m + \frac{1}{2}) \lambda \]

Destructive: \[ \Delta \phi = m \lambda \]

No n now because \( n=1 \) for the film.

I stress that you should avoid just memorizing these equations. Understand their source.
An example:

![Plano-convex lens and glass slide diagram]

At the contact point, there is an air gap of \( t \approx 0 \) thickness so reflective phase shift but \( t=0 \Rightarrow \) destructive.

Moving radially outward, the air gap grows \( \Rightarrow \) rings of constructive & destructive interference.

Called Newton's rings — yeah same guy. (He didn't get out much).

What if the light is not monochromatic? Then the constructive interference occurs at different places for different colors. (Destructive interference less clear because other colors fill in the previous gaps.)

Demo 84,27: Newton's rings.
Demo 84.27 Newton's rings

A curved piece of glass rests on a flat piece of glass and together they are mounted between adjustable metal plates (the black circular object with 3 thumbscrews). This assembly is then clamped to a base which is placed in front of a lens. Light from a projector is reflected off of the glass assembly and focused through the lens onto a screen. A red filter placed behind the lens makes the fringe lines more visible.
Another example of this is a soap film.

- A very thin film, $t \sim 0$, actually $t \ll \lambda$, gives just a phase shift at the air-soap interface.
  \[
  \Rightarrow \text{Reflected light is destructively interfered away. All transmitted.}
  \]

- As the film thickens, we get constructive interference for some colors at some spots and other colors in other spots.

Demo 84.24
Soap film interference

A bubble dispenser is dipped into a beaker of soap solution and then held in place in front of a lens. Light from a projector is reflected off of the soap film and focussed through the lens onto a screen. As the soap film slowly thins from top to bottom, rainbow bands appear on the screen.

Why is the dark region at the top? Gravity + Lens.

How thick is the soap film? \[ \lambda = \frac{500 \text{ nm}}{4} = 125 \text{ nm} \]
Conceptual question: A very thin soap film appears black as we just saw i.e., no reflection. But if you have an equally thin film of soap on a glass surface, it reflects brightly. Why?

\[
\begin{array}{ccc}
\text{air} & \text{soap} & \text{air} \\
\hline
n = 1 & n = 1.3 & n = 1 \\
\end{array}
\quad \begin{array}{cc}
\text{air} & \text{soap} & \text{glass} \\
\hline
n = 1 & n = 1.5 \\
\end{array}
\]

\[
\Delta \psi = \pi \\
\Delta \psi = 0 \\
\Delta \psi = \pi \\
\Delta \psi = \pi
\]

because \(n_{\text{glass}} > n_{\text{soap}}\)

So, you can use this to make reflective coatings on glass, or non-reflective, depending on the indices of refraction and the thickness of the coating.