Last time we looked at images from mirrors. We found that we could determine the position of the image just by tracing the reflection of the rays.

For a point source of light, all the reflected rays appear to originate from a point behind the mirror, and that image -- virtual image -- is the same distance behind the mirror as the object is in front of it.

\[ S = -S' \]

Where, you should recall the sign conventions, abbreviated as object distance is positive on the incoming side, image and radius of curvature is positive on the outgoing side.

Then we did the same for a curved mirror and found magnification:

\[ \frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \quad \text{and} \quad m = \frac{S'}{s} \]
Now, let's note what happens if the incoming rays are all parallel to the "optic axis."

All the reflected rays intersect at a point halfway between the center of curvature and the "vertex."

\[ \frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \implies \frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \]

\[ \Rightarrow s' = \frac{R}{2} \]

This is called the focal point because it is where distant objects (parallel rays) will have an image focused.

The "focal length" is \( f = \frac{R}{2} \).
So, we can rewrite the image-object relation as:

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

So, we can use this to calculate image location for any spherical mirror, but we can also use it to simplify graphing the image location with ray tracing, because there are a few rays that are particularly easy, called the "principal rays."

1). Along radial; bounces straight back
2). Through focal point, becomes parallel.
3). Parallel, reflects through f.
4). To vertex, equal angles.

\[s' < 0\]

\[m = \frac{-s'}{s} > 0\]
Now, let's do the same for convex mirrors.

As before,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

but both $s'$ and $f$ are < 0.

The image formed is virtual because the rays don't actually come from it, they just appear to emanate from that point.

Note, $M = \frac{-S'}{S}$ as before.
What would you get if you had light coming from all directions onto a convex mirror, aimed at its focal point?

Parallel rays reflected off the mirror.

Or you could turn it around and have parallel incoming rays, that reflect off going out as if from f.

\[ \frac{1}{\infty} + \frac{1}{s'} = \frac{1}{f} \quad \text{both} < 0 \]
Sample problem:

How could you make an image of yourself magnified by x10 and right side up?

Concave mirror: 10 = \(-\frac{s'}{s}\). \[\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}\]

If \(f = 10\ cm\), \[\frac{1}{s} + \frac{1}{(-10s)} = \frac{1}{10}\]

\[\Rightarrow \frac{10}{10s} - \frac{1}{10s} = \frac{1}{10}\]

\[\Rightarrow \frac{9}{10} = \frac{s}{10}\]

\[\Rightarrow s = 9\ cm\ from\ the\ mirror.\]

This is how a shaving mirror works.
Refraction at spherical surfaces.

We also get magnified images through refraction, but it is a bit more complicated because the angles are not all equal.

\[ \tan \alpha = \frac{h}{s + \delta} \quad \tan \psi = \frac{h}{R - s} \quad \tan \beta = \frac{h}{s' - \delta} \]

2. \( \theta_1 = \alpha + \psi \) because third angle must add to \( \pi \).

3. \( \psi = \beta + \theta_2 \) because \( \pi - \psi + \theta_2 = \pi - \beta \)
   \[ \Rightarrow \theta_2 = \psi - \beta \]

4. \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \)

Now, we have to make an approximation that \( \alpha \) is small. \( \Rightarrow \theta_1, \theta_2, \beta \) are also small. And \( \Rightarrow \delta \) is small. (Make \( \delta \to 0 \))
(4) becomes \( n_1 \theta_1 = n_2 \theta_2 \)

(2) \( \Rightarrow \quad n_1 \alpha + n_1 \psi = n_2 \theta_2 \)

\[ \theta_2 = \frac{n_1}{n_2} (\alpha + \psi) \]

(3) \( \Rightarrow \quad n_1 \psi - n_2 \beta = n_1 \alpha + n_1 \psi \)

\[ \Rightarrow \quad n_1 \alpha + n_2 \beta = (n_2 - n_1) \psi \]

(1) \( \Rightarrow \quad \alpha = \frac{h}{s} \quad \psi = \frac{h}{r} \quad \beta = \frac{h}{s} \)

So, \( \frac{n_1 h}{s} + \frac{n_2 h}{s'} = \frac{(n_2 - n_1) h}{R} \)

\[ \frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \quad \text{spherical refraction} \]
Now, let's look at an extended object.

\[ m = - \frac{n_1 s'}{n_2 s} \]

= lateral magnification.

(See text for derivation).

\[ s' > 0 \] on outgoing side, so inverted.