Nature and propagation of light - 1

We saw that EM waves are produced by an oscillating \( \vec{E} \) field -- and then they self-propagate.

We used an oscillating electric dipole as a simple example of a way to produce EM waves, such as a radio antenna. \( \oplus \rightarrow \bigoplus \)

Other oscillating charges should produce an oscillating \( \vec{E} \) as well. For example, suppose we had an electric dipole that was moving \( \oplus \bigoplus \) jiggling around.

Then \( \vec{E} \) at some points will change with time \( \Rightarrow \) EM waves.

Many molecules have a electric dipole moment (electrons pulled to one side, such as water)

If this water molecule oscillates around, it will give off EM waves.
How can you make a water molecule wiggle around? Heat it. \( \Rightarrow \) Thermal motion (and atomic vibrations, etc).

\[ \Rightarrow \text{EM waves.} \]

These tend to be higher frequency than radio waves: \( f = 10^{14} \text{ Hz} \) instead of \( 10^6 \text{ Hz} \) (or \( 10^9 \text{ Hz} \) for 802.11g).

Such EM waves are light. It is just like the other EM waves, but in a different range of frequency (or wavelength since \( c=2f \)).

We can see (with our eyes that is) light in the wavelength range between about

\[ \lambda = 400 \text{ nm} \text{ to } 700 \text{ nm} \]

blue \hspace{1cm} red \hspace{1cm} (compare to sound).

(actually violet)

There is, of course, a lot of light at other wavelengths to which we are oblivious.

Thermal radiation (the "heat light" above) is longer \( \lambda \) than red, so it is called infrared. (Unless things get very hot, then shorter wavelengths such as a candle).
Daylight is just thermal radiation from the surface of the sun. (It is "scattered" from the atmosphere and surroundings in ways we'll discuss later).

An incandescent light is thermal radiation from a hot wire.

Fluorescent lamps and LED's and lasers produce light through atomic processes that we'll discuss much later.

For now, let's note a couple things.

- LED's or lasers tend to be one color. But, an incandescent light is "white".

White is "all colors". All sorts of random motion.
Lasers (and some LEDs) can be coherent in various ways:

color, as we said
direction
Polarization $\mathbf{E}$
while the thermal oscillations produce $\mathbf{E}$ fields pointing every which way.

- Phase

So, to describe the white light from an ordinary light bulb we’d need:

$$\mathbf{E}(x,t) = \sum \mathbf{E}_i \cos(k_i x - \omega_i t + \phi_i)$$

and similar for $\mathbf{B}$.

Your eyes average all this to sense white. Hopelessly messy math—thankfully we don’t think about it.

And, to understand the nature and propagation of light, we definitely don’t want to think about such a mess. So, we simplify by considering single $\lambda$ light propagating in simple cosines.

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a light ray in one direction
When necessary, we can think about the wavefronts, but single "rays" is simpler and sufficient to describe most of the phenomena.

The first, and probably most fundamental issue is speed.

\[ \text{speed of light in vacuum} = C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \]

speed of light in another medium is \( \nu = \frac{1}{\sqrt{\varepsilon \mu}} < C \).

These speeds can be measured, and tabulated for different materials.

\[ n = \frac{C}{\nu} > 1 \] is called the index of refraction -- see later.

\[ n_{\text{vacuum}} = 1 \]

\[ n_{\text{water}} = 1.33 \text{ \Rightarrow "light boom"} \]

\[ n_{\text{glass}} = 1.54 \text{ \textit{Cherenkov radiation},} \]

\[ n_{\text{air}} = 1.0003 \]
This is called the index of refraction because as light passes between one medium and another, it refracts (bends) and the bend angle depends on n.

It is easy to see intuitively that light should bend this way.

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

(This is similar to why oblique waves turn toward shore).

Specifically, we can calculate this with the "law of refraction":

Snell's Law

\[ \theta_1 \text{ and } \theta_2 = \text{angles of incidence measured relative to normal.} \]

\[ n_1 < n_2 \Rightarrow \text{bends toward the normal.} \]
This goes both ways: if the light ray went right to left, it would bend away from the normal.

If $\theta_1 = 0$ then $\theta_2 = 0$. No bending then of course.

**Reflection**

If you recall our discussion of waves on a string, waves reflect when they reach a point of changing speed. Same with light.

The law of reflection is trivial.

$$\theta_1 = \theta_2$$

Demo 80.18: Reflection/Refraction in air-water interface.
You can see from the demo that at a certain point -- that is a certain angle, the light is totally reflected -- not refracted.

\[
\sin \Theta_2 = \frac{n_1}{n_2} \sin \Theta_1,
\]

If \( \Theta_2 = 90^\circ \) then it is right on the border between reflection & refraction.

For \( \Theta \) larger than that critical value, the light reflects.

\[
\Rightarrow 1 = \frac{n_1}{n_2} \sin \Theta_{\text{critical}}
\]

\[
\sin \Theta_{\text{crit}} = \frac{n_2}{n_1} = \text{critical angle for total internal reflection},
\]

For air-water.

\[
\sin \Theta_{\text{crit}} = \frac{1.0}{1.33} = 49^\circ \quad (\text{for air-glass} = 41^\circ)
\]
This is the basis for fiber optics.

Demo 80.27: Total internal reflection

Demo 80.30: Total internal reflection of laser beam in "fiber optics"
Conceptual question:

Which of these are physically possible refraction paths for a light ray?

\[ n_2 > n_1 \Rightarrow \text{bends toward normal.} \]

So (1) is OK. (3) is wrong. (2) is wrong. bending past the normal.

Conceptual question: Rank the following materials according to their index of refraction based on the shown light ray path.

\[ n_b > n_a \\
 n_b > n_c \\
 n_d > n_c \\
 n_d > n_a \\
 n_a > n_c \\
 n_d > n_b > n_a > n_c \]
Sample problem: Show that reflected rays A and B are parallel.

They are || if \( \theta_1 = \theta_3 \).

By the law of refraction (aka Snell's law):

\[
\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1, \quad \text{and} \quad \sin \theta_3 = \frac{n_2}{n_1} \sin \theta_2
\]

\[
\Rightarrow \sin \theta_3 = \frac{n_2}{n_1} \left( \frac{n_1}{n_2} \sin \theta_1 \right) = \sin \theta_1
\]

So \( \theta_3 = \theta_1 \) (since \( \theta_1 \) and \( \theta_3 \) are less than 90°).
Sample problem: A light ray moves between a medium with index of refraction $n_1$ and another with $n_2$. Show that Snell's law gives the shortest transit time between two points, A and B.

\[
t = t_1 + t_2 = \frac{l_1}{v_1} + \frac{l_2}{v_2} = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l-x)^2}}{v_2}
\]

To minimize the transit time, we take the derivative of $t$ with respect to the thing we can vary (like $\theta_1$). Actually, here, $x$ is easier, and both $\theta_1$ & $\theta_2$ depend on $x$.

\[
t = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l-x)^2}}{v_2}
\]

\[
\frac{\partial t}{\partial x} = \frac{x}{v_1} \left( \frac{h_1^2}{h_1^2 + x^2} \right)^{\frac{1}{2}} + \frac{x}{v_2} \left( \frac{h_2^2}{h_2^2 + (l-x)^2} \right)^{\frac{1}{2}} - \frac{x}{v_2} \left( \frac{h_2^2 + (l-x)^2}{h_2^2 + (l-x)^2} \right)^{\frac{1}{2}}
\]

chain rule
We find the \( x \) that minimizes \( t \) by setting this to zero:

\[
\frac{x}{v_1 \sqrt{h_1^2 + x^2}} = \frac{l-x}{v_2 \sqrt{h_2^2 + (l-x)^2}}
\]

\( v_1 = c/n_1 \) and \( v_2 = c/n_2 \), so

\[
\frac{x}{\sqrt{h_1^2 + x^2}} = \frac{l-x}{\sqrt{h_2^2 + (l-x)^2}}
\]

These can be related to \( \theta_1 \) and \( \theta_2 \):

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2
\]

\( \Rightarrow \) Snell's law is what minimizes transit time.

(Fermat's principle of least time).