Energy in electromagnetic waves.

Electromagnetic waves are perpendicular $\mathbf{E}$ and $\mathbf{B}$ propagating through space, and like all waves, they transmit energy.

Recall that we derived an expression for the energy stored in a charged capacitor. The energy density, energy per volume, is

$$U = \frac{1}{2} \varepsilon_0 \mathbf{E}^2$$

and the energy density in an inductor is

$$U = \frac{1}{2} \frac{1}{\mu_0} \mathbf{B}^2$$

Where in both cases the energy is stored in the fields themselves.

So an EM wave has energy in its $E$ and $B$ fields.

$$U = \frac{1}{2} \varepsilon_0 \mathbf{E}^2 + \frac{1}{2} \frac{1}{\mu_0} \mathbf{B}^2$$

$$\implies \mathbf{B} = \frac{\mathbf{E}}{c}$$

$$U = \frac{1}{2} \varepsilon_0 \mathbf{E}^2 + \frac{1}{2} \frac{1}{\mu_0} \frac{\mathbf{E}^2}{c^2} = \frac{\mathbf{E}^2}{2} \left( \varepsilon_0 + \frac{1}{\mu_0 c^2} \right)$$

but $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, so

$$U = \frac{1}{2} \mathbf{E}^2 \left( \varepsilon_0 + \frac{\mu_0 \varepsilon_0}{\mu_0} \right)$$

$$\implies U = \varepsilon_0 \mathbf{E}^2$$
As the wave propagates, this energy moves with it, and we are usually more interested in the power transmitted per area than in the energy density.

\[
\text{Power} = \frac{\text{Energy}}{\text{area}} = \frac{\text{Energy}}{\text{time}} \cdot \frac{\text{distance}}{\text{volume}}
\]

\[
dU = u \, dV = (\varepsilon_0 \, E^2) \, A \, c \, dt
\]

\[
\uparrow \text{volume} \quad \uparrow \text{length traveled} \\
\quad \text{energy density}
\]

\[
\Rightarrow \frac{dU}{dt} = \varepsilon_0 \, E^2 \, c \, A = \text{Power}
\]

\[
\frac{1}{2} \, \text{Power} = \varepsilon_0 \, c \, E^2 = \varepsilon_0 \, c^2 \, EB = \frac{EB}{\mu_0}
\]

There is a related vector quantity, called the Poynting vector (named after a guy named Poynting).

\[
\vec{S} = \frac{1}{\mu_0} \, \vec{E} \times \vec{B} \quad \Rightarrow \quad |\vec{S}| = \frac{EB}{\mu_0} \text{ since } \vec{E} \perp \vec{B}
\]

Gives the power/area in the direction of propagation.
We were most interested in intensity for sound waves. Recall that

\[ I = \frac{\text{Power}}{\text{Area}}. \]

So, is \( |\vec{S}| = I \)? Yes, sort of.

\[ I = |\vec{S}|_{\text{avg}} = \text{averaged } |\vec{S}| \]

where the average for a sinusoidal wave gives

\[ I = \frac{1}{2} \frac{E_{\text{max}} B_{\text{max}}}{\mu_0} \left( \frac{1}{\sqrt{2}} \right) \text{ for each of } E \& B. \]

\[ = \frac{1}{2} \varepsilon_0 c E_{\text{max}}^2 \]

Also recall that for a point source, the intensity decreased as we move away from the source

\[ I = \frac{\text{Power of source}}{4\pi r^2} \]

But this is only for a point source, not a source with a different shape.
I'd like to spend a little time discussing radio waves. I.e., how we can produce EM waves.

Although we did not need any charge to have an EM wave propagate, we do need charge to produce the varying E field to begin with.

Imagine that we do it as follows:

An electric dipole:
What are its field lines?

Now, suppose we switch the + and -.

E changes direction. So at this point we have a varying E that produces a varying B field (perpendicular to it) that causes an EM wave to propagate.
We can easily make such an oscillating electric dipole with a varying voltage.

Demo 76.12
Radio waves on oscilloscope

Demo 76.09
Radio waves light bulb.
So, what happens if I rotate the antennas so that the transmitter and receiver are not parallel, but instead perpendicular?

$\mathbf{S} = \frac{1}{4\pi} \mathbf{E} \times \mathbf{B}$ is perpendicular to $\mathbf{E}$

$\Rightarrow$ along propagation.

And, the orientation of $\mathbf{E}$ vs. $\mathbf{B}$ matters.

This $\mathbf{E}$ vs. $\mathbf{B}$ direction is called Polarization

(not to be confused with dielectric polarization.)

Normally light is randomly polarized, meaning that it has contributions with $\mathbf{E}$ pointing every which way. ($\mathbf{B}$ is always perpendicular to it though.)

Our radio waves are different since $\mathbf{E}$ is along the antenna -- the direction that the dipole is oscillating.

Demo

45°
Just as with other types of waves, we can have reflections that are at just the right position & frequency for the reflected waves to add constructively or destructively.

\[ \hat{E}(x, t) = E_{\text{max}} \cos(kx - wt) \hat{y} \]

\[ \hat{B}(x, t) = \frac{E_{\text{max}}}{c} \cos(kx - wt) \hat{z} \]

If we add two such waves, they interfere, just as for other waves, because it is simply due to trig arithmetic.

How do EM waves reflect? Suppose our radio wave hits a conducting sheet.

\[ \hat{E} = 0 \text{ in the conductor, ideally} \]

So, the \( \hat{E} \) in the wave must cause charges in the sheet to move. These charges produce an equal and opposite field \( \hat{E}_{\text{refl}} = 0 \).

\[ \text{Echarges} \]

\[ \text{Echarges acts like an oscillating dipole} \Rightarrow \text{reflected wave.} \]
Then, just as with sound, if the path difference is a multiple of $\frac{\lambda}{2}$ the waves cancel.

**DEMO**

\[ f = 150 \text{ MHz} \]
\[ \lambda = \frac{3 \times 10^8}{1.5 \times 10^5} = 2 \text{ m.} \]
\[ \Rightarrow \text{interference every 1 m.} \]

You can see an example of this in a microwave oven, if the food is not rotated, the standing waves leave cold spots. (Rings of unmelted cheese in my quesadilla).
Radiation Pressure

Not only do EM waves carry energy, they also carry momentum.

A beam of light shining on an object will transfer

\[ \frac{\Delta P}{\Delta t} = \frac{\Delta E/\Delta t}{C} \]

\[ \Delta P = \text{momentum transfer} \]

\[ \Delta E = \text{energy transfer} \]

Recall that \( \frac{\Delta P}{\Delta t} = \text{force}. \)

\[ F = \frac{1}{C} \text{Power} \]

Radiation pressure

\[ P_{\text{rad}} = \frac{E}{A} = \frac{1}{C} \frac{\text{Power}}{\text{Area}} = \frac{S_{\text{av}}}{C} = \frac{E}{C} \]

This has applications for solar sails, the solar wind, etc.
DEMO 76.18: speed of "light" through a coaxial cable.