Inductors

If two coils are near each other, a changing current in one induces a current in the other. We did an example of two solenoids.

\[ \Phi_B = \int \mathbf{B} \cdot dA \]

A current in the outer coil produces a magnetic field along the direction of the coil's axis. So, there is magnetic flux through the inner coil.

But, no EMF is induced if \( \Phi_B \) is constant.

A changing current produces a changing magnetic field, and hence a changing \( \Phi_B \).

Then,

\[ E = -N \frac{d\Phi_B}{dt} \]

- = Lenz's law

\( N \) = effect of \( \frac{d\Phi_B}{dt} \) on each loop of coil.

is not zero.

The inner coil has a current induced that pushes back—opposite \( \Phi_B \).
But what if we only have one coil.

Changing the flux, $\frac{d\Phi}{dt}$, also changes the flux in that coil's loops too. So, it should have an induced $\mathcal{E}$ as well. Right?

Yes. It does.

That is called self-inductance.

If there is initially no current flowing and I try to make a current flow through the coil, the changing current (and hence the changing $\mathcal{S}\Phi\cdot d\mathcal{A}$) induces an EMF that pushes back.

Who wins? Well, if I can apply sufficient external EMF (i.e., voltage) then the current will change.

Let's calculate this explicitly for the case of a solenoid and then generalize.
Inductance of a long solenoid.

If we have a long solenoid (approximate as infinite) we can calculate the EMF rather easily.

First, we need \( \vec{B} \) as a function of \( I \). \( \rightarrow \) \text{Ampere's Law}

\[ \oint \vec{B} \cdot d\vec{l} = B_0 l + 0 + 0 + 0 \]

By symmetry, \( \vec{B} \) must point along the axis, and be zero outside. So

\[ \mu_0 I_{ENC} = \oint \vec{B} \cdot d\vec{l} = B_0 l + 0 + 0 + 0 \]

\[ \Rightarrow B = \mu_0 \frac{I_n l}{l} = \mu_0 I_n \]

\( n = \text{loops/length} \)

Next, we need the magnetic flux.

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} = \int \mu_0 I_n \hat{z} \cdot d\vec{A} \]

\( \rightarrow \) Integral is trivial because the magnetic field is uniform.
Third, we need the time rate of change, $\frac{d\Phi_b}{dt}$.

$$E = -N \frac{d\Phi_b}{dt} = -N \mu_0 n \frac{d(I_A)}{dt}$$

$A$ is not changing in this case, only $I$. (Note that the textbook uses lowercase, $i$, for a changing current and uppercase, $I$, for a constant current. I won't do that because I think it is too much notation).

$$E = -\mu_0 N n A \frac{dI}{dt}$$

$N = \text{total # of turns}$  $n = \# \text{turns/length}$.

$$E = -\mu_0 n l n A \frac{dI}{dt}$$

$$E = -\mu_0 n^2 A \frac{dI}{dt}$$

This is a constant that depends only on the geometry of the solenoid.

So, define inductance for a solenoid as

$$L = \mu_0 n^2 A \ell$$
Like capacitance, $L$ depends on geometry:

$$C = \varepsilon_0 \frac{A}{d}$$
for a parallel plate capacitor.

Just geometry.

Also, like capacitance, if the solenoid is filled with a material other than vacuum, $\mu_0 \rightarrow \mu = K\mu_0$. $K$ can be huge for iron.

The units of inductance are Henrys, named after a guy that codiscovered inductance with Faraday.

$$1 \text{ H} = 1 \text{ T m}^2/\text{A}$$

A warning: $L = \mu_0 N^2 AL$ only applies to a long solenoid. In general it will be different. This is similar to $C = \frac{\varepsilon_0 A}{d}$ applying only to a large parallel plate capacitor. (definition)

The general equation for $C$ is $C = \frac{\varepsilon}{\kappa}$.

Similarly, the definition for inductance, $L$ is

$$L = N \frac{\Phi_B}{I}$$
relates flux and $I$, like $C$ relates $V$ and $Q$. 
We can see that this works for the special case of a solenoid because

\[ \Phi_B = \mu_0 I n A \]

\[ \Rightarrow L = N(\mu_0 I n A) / I = N \mu_0 n A = \mu_0 n^2 A L \]

Now, notice that

\[ E = - N \frac{d\Phi_B}{dt} = - \frac{d(LI)}{dt} \]

\[ = - L \frac{dI}{dt} \]

So, inductance \( L \) is just the connection between \( E \) and change in current.

\( E \) is related to the voltage across the inductor (details later).

\[ \Rightarrow V = L \frac{dI}{dt} \]

to be compared with \( V = IR \) and \( V = \frac{q}{C} \)

An inductor can be used as a circuit element where the voltage across it is dependent on the rate of change of the current through it.
Since inductors depend on changing current, they really are only useful in circuits with varying voltage and current.

⇒ AC circuits.

Such circuits are described in the text I can't cover that without dropping something else, more important.

Suffice it to say that the symbol for an inductor is

\[ \text{\includegraphics[width=2cm]{inductor.png}} \]

because that is exactly what it is, a coil of wire. (Often the wire is wrapped around an iron core to make \( \mu \) bigger.)
Transformers

An interesting application of inductance is transformers.

Suppose we take a "square donut" of iron and wrap coils around it.

If we run a current through the left wire, it produces a magnetic field pointed down. The field loops back around outside the "solenoid" made by the wire.

The magnetization induced in the iron makes the looping $\mathbf{B}$ field be confined to the iron.

So, the field—its flux—goes through the right coil... and it can induce an EMF there if the current is changing.

If we drive the left hand coil with a varying voltage, i.e. an $E$, then

$$E_{\text{left}} = N \frac{d\Phi_B}{dt}$$
and the same flux change is seen by the right hand coil.

\[ E_{\text{right}} = N_{\text{right}} \frac{d\Phi_B}{dt} \]

\[ \Rightarrow \frac{E_{\text{right}}}{N_{\text{right}}} = \frac{E_{\text{left}}}{N_{\text{left}}} \]

As before, converting \( E \rightarrow V \).

\[ V_{\text{right}} = \frac{N_{\text{right}}}{N_{\text{left}}} V_{\text{left}} \]

If we use more loops on one side than the other, the output voltage gets stepped up or down by the ratio, if--and only if--the voltage is varying.

This is why electrical power is distributed as AC, so it can be transformed to high voltage for transmission, then back down for use.
Energy storage in a magnetic field

Recall that we calculated last quarter that the energy stored in a capacitor is

\[ U = \frac{1}{2} CV^2 = \frac{1}{2} CE^2 d^2 \]

and we said that that energy is stored in the electric field.

Similarly, a magnetic field stores energy.

We can see that with an inductor,

\[ V = L \frac{dI}{dt} \]

so the power required to push a current in is

\[ P = IV = LI \frac{dI}{dt} \]

\[ \frac{dU}{dt} \Rightarrow \frac{dU}{dt} = LI \frac{dI}{dt} \]

\[ \Rightarrow dU = LI dI \]

\[ U = \int dU = \int LI dI = \frac{1}{2} LI^2 \]
So the energy per unit volume is

\[ u = \frac{U}{V} = ? \]

Well, for the special case of a solenoid,

\[ u = \frac{1}{2} L I^2 / (Al) = \frac{1}{2} \mu_0 n^2 A l I^2 / Al \]

\[ = \frac{1}{2} \mu_0 n^2 I^2 \]

\[ = \frac{B^2}{2 \mu_0} \]

\[ B = \mu_0 n I \]

\[ \Rightarrow \text{Energy stored in } B \text{ field.} \]
The energy stored in the B field comes out by inducing a current.

If we run a current through a solenoid, we get a B field. Once that becomes constant, suppose we turn off the current.

Then B decreases $\Rightarrow \frac{d\Phi_B}{dt}$ is negative.

$\Rightarrow E$ induced in the solenoid itself that tries to keep B constant.

If $I$ drops very fast, $\frac{d\Phi_B}{dt}$ is big and $E$ is big $\Rightarrow$ a large voltage.

DEMO 72.54 Inductor spark