Magnetic Induction 3

We have discussed induction as:

A change in magnetic flux induces an electromotive force that would cause a current that produces a magnetic flux that opposes the original change.

English is cumbersome, so this is easier to write as

\[ E = -\frac{d\Phi_B}{dt} \quad \text{where} \quad \Phi_B = \int B \cdot dA \]

But, make sure you understand what it means, "in your own words," so to speak.

We saw several examples of induction in action: a generator, the "ring flinger," etc. I want to spend today on a related phenomenon, eddy currents.
Eddy Currents

Suppose that we spin a bicycle wheel through a magnetic field;

As each spoke loop passes through the region with $\Phi_B$, a current is induced in the spokes. In which direction?

The left hand loop (pie shaped made of left spoke, sprocket, middle spoke, and rim) has a current producing $\bar{B}$ into page since it is losing flux that was into the page.

The right hand spoke loop is getting more $\Phi_B$ into the page induces a current that fights that making $\bar{B}$ out of the page.
If we thickened the spokes we'd have the same currents. In fact in the limit that the spokes fill the area with conductor, we still get currents circulating in the plane of the wheel as shown below.

Where this is the net of all the infinitely thin spokes we imagine in there.

These are called "eddy currents" because they look like the similarly named currents in fluid flow.

There are a few things to note here:

1. There must be some resistance in the metal, so current flowing in it will generate heat \((\text{Power} = I^2R)\).

By conservation of energy, then, the rotation of the wheel must slow.
We can see this more directly by calculating the force on the wheel.

Right in the "sweet spot" of the field the current is radially out.

\[ d\vec{F} = I d\vec{L} \times \vec{B} = I d\vec{r} \times \vec{B} \] points opposite to the rotation.

\[ \Rightarrow \text{Magnetic brakes---with no lining to wear out.} \]

2). We could actually use this to produce current that recharges a battery when we apply the brakes.

\[ \Rightarrow \text{Less energy wasted as heat} \]
\[ \Rightarrow \text{Better gas mileage} \]

That is one of the tricks used in hybrid, gas-electric, cars.
Some eddy current demos.

So it acts as a magnetic brake. What happens if I move the magnet over a stationary conductor? Same forces as above -- relative motion -- so wheel rotates!

Here we have a magnet in contact with the wheel. Again, relative motion induces eddy currents.

These eddy currents produce an opposite magnetic field, so the magnet levitates.
Demo 72.27
Eddy current tube

As the magnet falls, the flux in a ring shaped segment of the tube 1 decreases as the magnet moves down.

\[ \frac{d\Phi_B}{dt} \to \mathcal{E} \]

Lenz's law ⇒ direction of induced eddy currents are \( \mathcal{S} \)
\[ \mathcal{S} = \mathcal{D}_N \]

\( \overline{N} \) attracts the magnet back up.

Ring 2 has \( \Phi_B \) increasing upward so the induced current tries to make more \( \Phi_B \) down.

\[ \mathcal{V}_N = \mathcal{S}_N \]

This repels the magnet.

Both affects slow the magnets fall.
Same result if \( \overline{N}_S \) changed to \( \mathcal{S} \).
3). These eddy currents produce a magnetic field, of course, as any current does. We could detect that magnetic field. How? By putting a loop nearby and having it induce an $E$. This is the principle behind a metal detector -- it detects eddy currents.

No voltage here unless there is a changing flux. If a conducting object moves by the inner loop, it induces an eddy current and that current produces a magnetic field that induces an EMF in the outer loop.

The "receiver" coil can also be on the other side and you can pulse the current in the main coil and "detect" it in the opposite coil. Intervening metal causes a smaller signal because the eddy currents oppose the signal.
Conceptual questions:

A long wire carries a current $I$.
Three wire loops are placed near the wire, but not touching it, as shown.

Assume that the loops are far from each other so they have no mutual effects.

Rank the loops according to the current induced in them.

First, if $I$ is constant, then $\vec{B}$ is constant, $\Phi_B$ is constant, and $E=0$ for all loops.

OK, then, what if $I$ is increasing with time?

A: Since the loop has the same area on both sides of the wire, as many $B$ field lines come out of the top half as go into the bottom half, so $\Phi_B = 0$, always.

So

$E = 0$ for loop A
B: The current in the wire causes $\vec{B}$ to come up out of the page through the loop. As $I$ increases, $|\vec{B}|$ increases and $|\Phi_B|$ increases.

C: The current in the wire causes $\vec{B}$ to go down into the page through the loop. As $I$ increases, $|\vec{B}|$ increases and $|\Phi_B|$ increases.

$|\Phi_B|$ is bigger for loop C than for loop B because C has a larger area.

So $E$ is larger magnitude for C than for B.

What are the directions of the induced currents (or EMFs)?

B: $\vec{B}$ is increasing out of the page, so current flows clockwise in the loop to make an oppositely directed field.

C: $\vec{B}$ is increasing into the page, so current flows counterclockwise to make an oppositely directed field.
Conceptual question:

A constant current flows through a long wire.
A conducting loop near it is pulled away.
What direction is the induced current in the loop?

\[ \phi_B = \int S \cdot dB \] in the loop is changing because \( B \) is decreasing as we move away from the wire.

The induced current is in a direction that opposes this change (decrease) in \( \phi_B \) by adding more \( \phi_B \) out of the page.
\[ \Rightarrow \text{The induced current is counter-clockwise} \]

If time allows, describe magnetic induction in an electric guitar's pickups, and various ways to add new distortion effects.
Induced Electric Fields

If an induced EMF causes a current to flow, there must be an electric field pushing the charges.

⇒ A changing magnetic field produces an electric field.

Suppose, for example, that we have a changing magnetic field from a solenoid.

Assume that $B$ is increasing at a rate $dB/dt$, $\Rightarrow \vec{E}$ such that $I$ is counter clockwise.

That means there is an $\vec{E}$ field "pushing charge" around the ring.

By symmetry, $\vec{E}$ must be tangent to the circle and the same everywhere.

A charge pushed around the circle by $\vec{E}$ has a total work, $W$, done on it.
\[ W = \int \vec{F} \cdot d\vec{l} = (q_0 \vec{E}) 2\pi r = q_0 \vec{E} 2\pi r \]

\[ \vec{F} = q_0 \vec{E} \text{ and } d\vec{l} \text{ always parallel.} \]

\[ = q_0 \int \vec{E} \cdot d\vec{l} \]

\[ \frac{W}{q_0} = \int \vec{E} \cdot d\vec{l} \]

But \( \frac{W}{q_0} = E \) since \( V = \frac{W}{q_0} = \frac{\text{energy}}{\text{charge}} \)

\[ \Rightarrow \text{Faraday's law can also be written as} \]

\[ \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]

This is true for any closed path, the line integral of \( E \) around the path, is the change in magnetic flux. There does not need to be a conductor there. If it were an insulator, then it could become polarized by this \( E \) even though \( I = 0 \).
Another way to see this induced electric field is with a moving conductor.

\[
\begin{align*}
\text{Suppose a rod of length } l \\
\text{moves at constant velocity} \\
\text{perpendicular to a magnetic field (into the page).}
\end{align*}
\]

Each of the free charges in the conductor are also moving with velocity \( \overrightarrow{v} \). So they experience a force

\[
\overrightarrow{F} = q \overrightarrow{v} \times \overrightarrow{B} = qvB
\]

that pushes the positive charges up the rod and the negative charges down the rod.

As the ⚫ and ⚫ charges move apart, they produce an electric field, and that field causes an opposing force, \( F = qE \). The charges move until the magnetic and electric forces balance.

\[
\Rightarrow E = vB \hspace{1cm} \text{(magnitudes only, directions are } l) \hspace{1cm}
\]

So,

\[
V = \int \overrightarrow{E} \cdot d\overrightarrow{l} \text{ gives } EMF \hspace{1cm} E = vLB
\]

If it were an insulator instead, then polarized.