Measuring index of refraction

As a review of recent topics, let's think about how we could measure the index of refraction of some unknown glass.

For example, if someone claims that they have a slab of diamond, but you think it is just quartz glass, how could you determine for sure? Diamond has \( n = 2.417 \) while glass is more like \( n = 1.5 \).

The most direct way is with Snell's law.

\[ \frac{n_\text{air}}{\sin \theta_1} = \frac{n}{\sin \theta_2} \]

\[ S = d + \tan \theta_2 \]
\[ a = d + \tan \theta_1 \]

And we can determine \( a \) and \( S \) by measuring \( x \) on a screen. \( x = a - S \) and \( \tan \theta_1 = \frac{h}{L} \)

This is conceptually simple, but not precise because we are trying to measure angles.
A cleaner way would be to use the slab as a lens.

Light from an object going into the lens' $1^{st}$ surface gives an image at $S_{im1}$ where

$$\frac{1}{S_{obj}} + \frac{n}{S_{im1}} = \frac{n-1}{R} \rightarrow 0$$

$$\Rightarrow S_{im1} = -n S_{obj}$$

Suppose we did this with a point of light as the object (say a laser spot), as shown below.

Draw one ray to find image 1, then image 2.

Then we use im1 as an object for the second surface...
So,
\[ \frac{n}{|s_{im1} + d|} + \frac{1}{s_{im2}} = \frac{1-n}{R} = 0 \]

\[ \frac{n}{n s_{obj} + d} = -\frac{1}{s_{im2}} \]

Note that \( s_{im2} < 0 \)

\[-s_{im2} = \frac{n s_{obj} + d}{n} = s_{obj} + \frac{d}{n} \]

For ease of measurement, we could move the slab until the final image appears to be at the corner of the slab, so, \( s_{imx} = -d \)

\[ \Rightarrow \quad -(-d) = s_{obj} + \frac{d}{n} \]

\[ \Rightarrow \quad d - s_{obj} = \frac{d}{n} \]

\[ n = \frac{d}{d - s_{obj}} \]

Just measure \( s_{obj} \) when the image appears to be at the corner and calculate \( n \).
We could measure \( n \) directly from its definition related to speed, \( n = \frac{c}{v} \).

Send a short laser pulse and measure the time delay.

\[ \text{Detector} \]

\[
\begin{align*}
\text{Laser} & \quad \text{Partial Mirror} \quad \text{Mirror} \\
& \quad \quad \quad \text{Detector}
\end{align*}
\]

Would see two pulses, one from the short path, then a second from the long path. \[ t_{\text{air}} \]

\[ t_{\text{air}} = \frac{L}{c} \] with \( L \) being the extra path length.

Then place the glass in the long path, and it will delay the longer path.

\[
\begin{align*}
t_{\text{glass}} &= \frac{L-d}{c} + \frac{d}{v} \\
&= \frac{L}{c} - \frac{d}{c} + \frac{d}{c/n} \\
&= t_{\text{air}} + \frac{d}{c}(n-1)
\end{align*}
\]
Note that this will be hard to measure. If \( d = 1 \text{ cm} \) and \( n = \sqrt{2} \), then

\[
t_{\text{glass}} - t_{\text{air}} = \frac{d}{c} (n - 1) = \frac{10^{-2} \text{ m}}{3 \times 10^8 \text{ m/s}}
\]

\[
= 0.3 \times 10^{-10} \text{ s}
\]

which is a very short time!

An interferometer would be better.

Path 1 has a phase shift of \( \pi + \pi + \frac{2 \pi}{\lambda} l_1 \).

Path 2 has a phase shift of \( \pi + \pi + \frac{2 \pi}{\lambda} l_2 \).

So, the net phase difference is \( 2\pi \left( \frac{l_1}{\lambda} - \frac{l_2}{\lambda} \right) \).

If we put the slab into the path 2, it changes \( \pi \) for part of the path...
\[ V = \lambda f \Rightarrow \lambda_g = \frac{V}{f} = \frac{c/n}{f} = \frac{c/n}{c/\lambda_{air}} \]

\[ \lambda_g = \frac{1}{n} \lambda_{air} \]

So, the phase shift along path 2 changes by

\[ \Delta \psi = 2\pi \left( -\frac{d}{\lambda_{air}} + \frac{d}{\lambda_g} \right) \]

\[ \Delta \psi = 2\pi d \left( \frac{1}{\lambda_g} - \frac{1}{\lambda_{air}} \right) = 2\pi d \left( \frac{n}{\lambda_{air}} - \frac{1}{\lambda_{air}} \right) \]

\[ = \frac{2\pi d}{\lambda_{air}} (n-1) \]

Again, this works in principle, but if \( d \gg \lambda_{air} \) it is hard to measure because \( \Delta \psi \gg 2\pi. \)
An important point to remember in all of this, though is that the index of refraction can be a function of the wavelength of the light.

This is what we called dispersion.

Recall how dispersion is responsible for rainbows and the spectrum from a prism.