Waves

- What is a wave? A travelling disturbance.
  - A transverse wave has the disturbance direction transverse to the propagation direction.
  - A longitudinal wave has the disturbance along the propagation direction.

Described as $y(x,t)$ if transverse, where $y(x,t) = f(x \pm vt)$.

If periodic,

$$y(x,t) = A \cos \left[ kx - \omega t \right]$$

$$= A \cos \left[ 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \right]$$

where $\lambda = \text{wavelength} = \text{distance from one peak to next}$

$T = \text{period} = \text{time from one peak to next}$

$$\omega = 2\pi f,$$  $k = \frac{2\pi}{\lambda},$  $v = \lambda f = \frac{x}{T}$

$v = \sqrt{\frac{E}{\mu}}$ for a string  $v = \sqrt{\frac{B}{\rho}}$ for pressure wave

$\approx \sqrt{\frac{\text{restoring force}}{\text{inertia}}}$
Waves transfer energy through the medium

\[ \langle P \rangle = \frac{1}{2} \sqrt{4\pi F} \omega^2 A^2 \quad \text{needn't memorize} \]

and, more importantly, intensity is

\[ I = \frac{P}{\text{Area}} \quad \text{so} \quad \frac{I_1}{I_2} = \frac{P/4\pi r_1^2}{P/4\pi r_2^2} = \left( \frac{r_2}{r_1} \right)^2 \]

Intensity level, in dB,

\[ \beta = (10 \text{ dB}) \log \frac{I}{I_0} \quad I_0 = 10^{-12} \text{ W/m}^2 \]

Waves reflect (echo) from boundaries, or changes in wave speed.

Reflected wave passing back through original wave superposes with it (displacements add algebraically), and the two waves can interfere, and produce a standing wave if the length is just right.

\[ \nu = f \lambda \]

\[ \Rightarrow f = \frac{\nu}{2L} \]

\[ L = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \Rightarrow f = n \frac{\nu}{2L} \]

\[ \Rightarrow 1, 2, 3, \ldots \]
Sound is just longitudinal pressure waves. Or we can think of the air (or medium) displaced for $\pi$ back.

Just as reflection & superposition gives standing waves on a string, we get standing waves in a pipe.

Open

- antinode for displacement = pressure node

\[ \Rightarrow L = \frac{\pi}{2} \]

\[ \Rightarrow f_1 = \frac{\nu}{2L} \]

\[ f_n = \frac{\nu}{2L} \quad n = 1, 2, 3, \ldots \]

Stopped

- displacement node

\[ f_n = \frac{\nu}{4L} \quad n = 1, 3, 5, \ldots \]

Similarly, sound waves with the same $f$ can interfere if they have different pathlengths, (or phase shifts).

Destructive if $\Delta L = \frac{n\pi}{2}$

Constructive if $\Delta L = n\pi$

\[ \sim \]
Adding waves of different frequency also causes interference, but it varies in time, not just position.

\[ a(t) + b(t) = 2A \sin \frac{\omega_a - \omega_b}{2} t \cos \frac{\omega_a + \omega_b}{2} t \]

Beat frequency \( f_{\text{beat}} = f_a - f_b \).

Doppler shift. Wavefronts squeezed together in front of moving source, spread out behind.

\[ \Rightarrow f \text{ increases} \quad \Rightarrow f \text{ decreases}. \]

A moving listener crosses wave peaks so \( f \) also increases or decreases.

\[ f_L = \frac{V + U}{V + U_s} f_0 \]

with signs that give a sensible answer.

Supersonic source \( \Rightarrow \) shock wave.
Electric force

\[ \vec{F} = \kappa \frac{q_1 q_2}{r^2} \hat{r} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \]

\( \approx 9 \times 10^9 \text{ N m}^2 \text{c}^2 \) = big.

But only true for point charges. Any extended object is a sum of forces.

\[ \text{Need to add up } \int \text{ the infinitesimal pieces.} \]

But, keep in mind that \( \vec{F} \) is a vector.

Electric field is force per charge.

\[ \vec{F} = q \vec{E} \]

and can be similarly calculated, but we can also think about the electric field existing independent of the second charge. Electric field lines show the direction of force on a positive charge.
Calculating $\hat{E}$ can be made easier with Gauss' law.

$$d\Phi = \vec{E} \cdot d\vec{A}$$

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{Enclosed}}}{\varepsilon_0}$$

where $\oint$ is over an enclosing gaussian surface.

True for any surface, so we can choose the easiest one.

Example: Charged sphere:

$$\frac{q}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A} = \oint E dA = E \oint dA = E \cdot 4\pi r^2$$

$$E = \frac{q}{4\pi \varepsilon_0 r^2}$$

Easy because of spherical symmetry.

Could we use a cube? A cylinder? An offset sphere?

What is $\Phi$ through a cube offset?
Used gauss' law to see that all charge lies on the surface of a conductor.

First, what is a conductor & insulator.

**Conductor =** free charges move easily in $E$ field, as they move apart, they balance the field.

**Insulator =** charges can't move, they can only be polarized.

So in a conductor $E = 0$, or else $q$ moves to make it so.

$E = 0 \Rightarrow q$ outside $\Rightarrow$ must be on surface.

If an insulator, $q$ stays where we put it... so have to say where. We only dealt with uniform distributions for simplicity.
Example: Insulating sphere \frac{1}{2} conducting shell.

What is \( E \) everywhere?

If \( E = 0 \) is conductor, but \( +q \) is enclosed there must be a \(-q\) induced on inner surface.

\[ \Rightarrow \text{Another} \ +q \ \text{induced on outer surface.} \]

\[ \Rightarrow \text{Total charge} = 0. \]

We did some homework problems on this charge induction.

Example charged comb attracting uncharged paper bits, by polarization.
Electric Potential energy

\[ V = \oint \vec{E} \cdot d\vec{l} = \oint \vec{E} \cdot dl \]

where integral is from where we are to where we define to be 0.

\[ = \text{Just the work we could potentially do.} \]

As before, divide of the \( q \), so

Electric Potential \( V = \oint \vec{E} \cdot d\vec{l} = \text{volts} = \frac{J}{C} \)

No specific zero point, only differences matter.

Should be able to calculate \( U \) and \( V \).

\( U \) adds like \( \vec{F} \) and \( \vec{E} \)

\[ U_{\text{point charge}} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r} \]

\[ V_{\text{point charge}} = \frac{1}{4\pi \varepsilon_0} \frac{q}{r} \]

\[ \Rightarrow V = \frac{1}{4\pi \varepsilon_0} \int \frac{dq}{r} \quad \Rightarrow \vec{E} = -\nabla V \]

\[ \Rightarrow \vec{E} = 0 \text{ means } V \text{ constant.} \]
Capacitors:

\[ C = \frac{Q}{V} \Rightarrow \text{how much potential necessary to put } Q \text{ on it.} \]

Deriving expression for a parallel plate capacitor is a good example.

\[ C = \frac{Q}{V} = \frac{Q}{\int E \cdot dA} \]

\[ E \text{ by Gauss' law.} \]

Top:

\[ \oint E \cdot dA = EA + EA + 0 \]

\[ \Rightarrow \frac{\sigma A}{\varepsilon_0} = 2EA \Rightarrow E = \frac{\sigma}{2\varepsilon_0} \]

Same for bottom, so \[ E_{+0} = \frac{\sigma}{\varepsilon_0} \]

\[ V = \oint E \cdot dA = \frac{\sigma A}{\varepsilon_0} \]

\[ \Rightarrow C = \frac{\sigma A}{\varepsilon_0 \frac{d}{A}} = \varepsilon_0 \frac{A}{d} \]
Combining two capacitors in parallel adds their capacitances.

\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \]

same \( V \) across them

In series

\[ \frac{1}{C} = \frac{1}{C_1} - \frac{1}{C_2} \]

Induced charge here, so in series they have the same charge, not voltage.

If there were a dielectric between the plates, then polarization reduces the field so more \( \varepsilon \) for same voltage.

\[ \Rightarrow \text{higher } C \]

\[ C = kC_0 \]

\[ \Rightarrow \text{dielectric constant } \varepsilon > 1 \]

\[ \Rightarrow \varepsilon_0 \rightarrow \varepsilon \]
Current:

\[ I = \frac{dq}{dt} = \text{flow of charge, pushed by potential.} \]

\[ J = \frac{I}{A}, \quad \frac{j}{J} = \frac{E}{\rho} \quad \text{resistivity = material property.} \]

\[ V = IR, \quad \frac{V}{A} = \frac{V}{S} \Rightarrow R = \frac{\rho L}{A} \]

Power = energy per time.

\[ = \frac{V \cdot \text{charge}}{\text{time}} = VI \]