DC Circuits

We have studied several things about the electric force, and finally come to practical applications: voltage, current, and new circuits.

Some of you will have already learned this in EE. So, just do our examples on your own.

Let me begin with a bit of introduction/review.

Current is flow of charge in a conductor under the influence of an electric potential

\[ I = \frac{V}{R} \]

The current magnitude is the voltage divided by the resistance.

There are various sources of potential (voltage). Recall that we calculated the potential for a charge distribution, so we can get a voltage from a "pile of charge." (The amount of voltage we get is determined by the capacitance, \( V = \frac{Q}{C} \)).

But that voltage won't last too long, so it won't give a constant current.
A battery gives a more constant voltage because it is set by the EMF of the battery—\textit{the chemical potential}. Similarly, a solar cell has an EMF set by the intrinsic energy level in the semiconductor junction.

\[ \Rightarrow \text{Constant EMF.} \]

A real battery however has some non-zero internal resistance, which we've called $R_{\text{int}}$.

So a simple circuit could be

\[ \text{which is just a heater.} \]

With what power does it heat?

\[ P = IV \]

What is $I$?

This leads to today's topic: analysis of circuits.

I'll start with a couple examples and then formalize it.
In our simple circuit we can figure out $I$ intuitively.

- Charge is conserved so whatever $I$ flows through $R_{\text{int}}$ must also flow through $R$.

- $V = IR$ (where $V$ means the voltage difference, also called the voltage drop, across any thing)

  So, $V$ after $R_{\text{int}}$ is $1.5 - IR_{\text{int}}$.

  And $V$ after $R$ is $1.5 - IR_{\text{int}} - IR = 0$ back at battery.

  $\Rightarrow 1.5 = IR_{\text{int}} + IR$

  $\Rightarrow I = \frac{1.5}{R_{\text{int}} + R}$

  $\Rightarrow I = \frac{1.5}{R_{\text{int}} + R}$

  and $P = IV = \frac{(1.5)^2}{R_{\text{int}} + R} = \frac{V^2}{R_{\text{total}}}$

If $R = 1\Omega$ and $R_{\text{int}} \to 0$, then $P = 2\text{W}$, not much heat.
Let's do some more examples to gain familiarity. In the following I will assume that \( r_{\text{int}} = 0 \), a pretty good approximation for fresh batteries.

What is the voltage here?

First, note that the actual voltage is only defined as a difference, so I have to define a zero point somewhere. I'll use the negative terminal of the battery. So,

I is same through both resistors because charge is conserved.

\[ \Rightarrow \text{Voltage dropped across each is the same.} \]

And voltage at that point is \( \frac{V}{2} \).

This is intuitively obvious if you think about "voltage drops." Examples.
Kirchoff's rules

We've now seen Kirchoff's rules as just obvious results of charge conservation. But, let's formally state them.

\[ \sum \text{Voltage Drops} = 0 \] around a closed loop

which is loosely written as \[ \sum V = 0. \]

And,

\[ \sum I = 0 \] into any point, where this sum has to take into account the signs of the current.

E.g.,

\[ \begin{align*}
\text{At a) } \sum I &= 0 \quad \text{means} \\
&\quad \rightarrow \text{ from left and} \\
&\quad \leftarrow \text{ in from right,} \\
&\quad \text{similarly for (b) and (c).}
\end{align*} \]

and

\[ V_{ab} + V_{bc} + V_{ca} = 0 \]
\[ IR + IR - V = 0 \]

\[ \Rightarrow I = \frac{V}{2R} \]
Let's use these rules to determine how resistors combine in series and parallel.

\[ V = V_1 + V_2 = IR_1 + IR_2 \]
\[ \Downarrow \]
\[ IR_{\text{EFF}} = IR_1 + IR_2 \]
\[ \therefore R_{\text{EFF}} = R_1 + R_2 \]

\[ V = I_1 R_1 = I_2 R_2 = IR_{\text{EFF}} \]
\[ I = I_1 + I_2 \]
\[ \Rightarrow \frac{V}{R_{\text{EFF}}} = \frac{V}{R_1} + \frac{V}{R_2} \]
\[ \Rightarrow \frac{1}{R_{\text{EFF}}} = \frac{1}{R_1} + \frac{1}{R_2} \]

Note similarity to combining capacitances, but
\[ q = CV \quad \& \quad I = \frac{V}{R} \]
An example: What is the equivalent resistance?

\[ \frac{1}{\frac{1}{5} + \frac{10}{3} + \frac{15}{3}} = \frac{3}{15} + \frac{3}{25} \]
What if we add batteries?

What is \( I \)?

By Kirchoff's rules, the total voltage is zero around the loop, so call this point zero.

\[
V_2 = IR_1 + V_1 - IR_2 - IR_3 = 0
\]

\[
V_1 + V_2 = I(R_1 + R_2 + R_3)
\]

\[
I = \frac{V_1 + V_2}{R_1 + R_2 + R_3}
\]

Where \( R_1 \) is didn't matter in this case, just equal \( I \) s.t. \( \Sigma V = 0 \).
How about this?

\[ V_2 - V_1 = IR_1 - IR_2 = 0 \]

\[ V_2 - V_1 = I(R_1 + R_2) \]

\[ I = \frac{V_2 - V_1}{R_1 + R_2} \]

The batteries may partially cancel if reversed polarity.

How about this?

\[ \sum V = 0 = V_1 - I_1R_1 + I_2R_2 - V_2 = 0 \text{ starting at } A \text{ going clockwise.} \]

\[ \sum V = 0 = V_1 - I_1R_1 + I_2R_2 = 0 \]

\[ \sum V = 0 = V_2 - I_2R_2 + I_3R_3 \]

\[ I_1 + I_3 + I_2 = 0 \text{ considering junction a point } B. \]

Note that this implies that one of the currents must be negative i.e., opposite the direction assumed.

Four equations w/ 3 unknowns, just math now.
Conceptual question

which side of the battery is $+$ and which is $-$?

Conceptual question

What is $V_{AB}$ when the switch is open? 10 V

When the switch is closed? 0 V.

If I put a 10Ω resistor in parallel with the battery, how would the answer change? It wouldn't.
Short circuit a battery:

\[ \frac{1}{\text{Ohm}} = 0 \]

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