Last time we talked about electric current, which is the motion of charge. Let's review the important points there:

- Charge is caused to move by an electric field.

- Only mobile charges move; conductors have mobile charges, insulators have immobile charges.

- We can describe current microscopically in terms of moving charges as

\[ I = \frac{dQ}{dt} = neA \nu_d \]

\[ \frac{\text{volume per time}}{\text{charge per volume}} \]

\( N \) = charge carrier density is enormous, so \( \nu_d \) is small.

Units = Coulomb/second = Amps

Since we don't always know \( A \), we can define a current density

\[ \vec{J} = \frac{I}{A} = ne \vec{\nu}_d \]

Units = Amps/m²
- The direction of \( \vec{J} \) (which is a vector) is along the electric field, and since positive charges move in the direction of \( \vec{E} \) that means current is thought of as moving positive charges by convention. Even if it is actually electrons that are moving.

- \( I \) is not a vector, since the area that it flows through can wiggle around the path of the conductor. But \( I \) has a sign: positive \( I \) is the direction that positive charges flow.

- As the charges flow, they scatter off (i.e. bang into) each other and the fixed molecules. This converts some of the energy they gained from \( \vec{E} \) into heat. This resists the flow of charge. Resistance is defined as

\[
R = \frac{V}{I} \quad \text{or} \quad V = IR
\]

A larger \( R \) means more potential is required to get a certain current.

Units = \( \frac{\text{Volts}}{\text{Amp}} \) = Ohms = \( \Omega \)
A microscopic equivalent is

\[ \rho = \frac{E}{J} \]

equals resistivity

which is useful because \( \rho \) is a property of the material regardless of its shape.

\[ \rho = 1.7 \times 10^{-8} \Omega \cdot m \] for copper

\[ \rho = 10^{10} \Omega \cdot m \] for glass \( \Rightarrow \) insulator

Since \( \rho \) is a property of the material, we can use it and geometry to determine \( R \) for some shape.

\[ R = \frac{V}{I} = \frac{E \cdot L}{J \cdot A} = \frac{\rho \cdot L}{A} \]

This ratio makes intuitive sense.

- Ohm's law is an empirical observation that \( R \) and \( \rho \) are approximately constant; they don't depend on \( V \) or \( I \), for many metals. But not all conductors.
Now, I want to describe power in electric current flow.

As I said, resistance comes about because the charge carriers, electrons, bang into molecules and transfer any energy gained from $E$ into molecular motion -- or vibration -- which is heat.

It requires potential energy to get them -- the charges -- moving and that goes into heat ($V/g$ is tiny). So, how much power goes into heat?

Well $P = IV$ I can just say it, but it is better to understand it.

The potential energy is $U = Vg$, so

$$dU = Vdg$$

$$\frac{dU}{dt} = V \frac{dg}{dt} \implies P = V I$$

Just follows simply from the definition of $V$ and $I$. 
Note that resistance never came into this. Rather, the power is just moving charge through V, so it is true in general.

\[ P = VI \] is power for any current flowing through a potential difference V.

So, if you have a 1kW hair dryer, how much current does it use?

\[ 1 \text{ kW} = (110 \text{ V}) I \Rightarrow I \approx 9 \text{ Amps}. \]

(Most circuit breakers can only handle 15A, meaning they are designed to shut off the circuit if I exceeds 15Amps. So if you run a few hair dryers at once, the breaker will blow).

In the special case of a resistor,

\[ P = IV \]

can be rewritten as

\[ P = IV = I(\frac{V}{R}) = I^2R \]

or

\[ P = IV = \frac{(V^2)}{R} = \frac{V^2}{R} \]
Example: A toaster produces heat by running current through a long thin wire made of Nichrome. This is a nickel-chromium-iron alloy chosen because it has a fairly large resistivity $\rho$.

$$\rho = 100 \times 10^{-8} \text{ Ohm} \cdot \text{m}$$

If you have a wire of length $L = 56\text{m}$ and diameter $d = 1\text{mm}$, what power is dissipated in it?

$$P = \frac{V^2}{R} \quad V = 110 \text{ Volts from the wall}$$

Just need to know $R$.

$$R = \rho \frac{L}{A} = 100 \times 10^{-8} \frac{56}{\pi (0.001)^2} = \frac{56}{\pi} \times 10^6 \times 100 \times 10^{-8}$$

$$= \frac{3.24}{\pi} \approx 70 \ \Omega$$

$$\Rightarrow P = \frac{(110)^2}{70} = \frac{12100}{70} \approx 170 \text{ W}$$

What if you cut the wire in half?

$$P = \frac{12100}{35} = 340 \ \text{W} \Rightarrow \text{can burn toast faster!}$$

A problem with this is that the power/length quadruples and the nichrome can melt. (This is also why you want a long wire, not a tiny 70 $\Omega$ resistor.)
Sample problem: Power distribution.

Suppose you are trying to supply power to all of Santa Barbara from LA with a very long extension cord. Will this work?

SB uses about 100,000 people * 1 kW/person = 100 MW of power. What current is required?

\[ P = IV \Rightarrow I = \frac{P}{V} = \frac{100 \text{ MW}}{110 \text{ V}} = 1 \text{ MAmp}. \]

How much heat is dissipated in the extension cord?

\[ R = \frac{1}{A} = 1.7 \times 10^{-8} \text{ \Omega.m} \frac{200 \text{ km}}{\pi (10^{-2})^2} \]

\[ = \frac{3 \times 10^{-6}}{\pi} \times 10^4 \times 10^3 = 10 \text{ ohm, small resistance actually!} \]

So,

\[ P_{\text{heat}} = I^2 R = (10^6)^2 (10) = 10^{12} \to 10^{13} = 10 \text{ TW}. \]

10 million times as much heat lost as useful power delivered.

It would actually melt the copper! How to get around this?

Use \( V = 100,000 \) volts instead of 110 V.

\[ P = IV \Rightarrow I = \frac{100 \text{ MW}}{100 \text{ KV}} = 1000 \text{ A}. \]

\[ P_{\text{heat}} = I^2 R = (10^3)^2 (10) = 10^7 = 10 \text{ MW, relatively small.} \]

(Works, just don't touch the wires.)
A related example is jumper cables. They are usually very thick cables to reduce $R$. Why?

Starting a car requires a lot of power, about a KW, but we only have a 12V battery.

$$\Rightarrow I = \frac{P}{V} = \frac{1000}{12} \approx 100 \text{ Amps.}$$

$R$ of the cables $\Rightarrow R = \frac{\rho L}{A} = 1.7 \times 10^{-8} \frac{2m}{\pi (10^{-3})^2} = 10^{-4} \Omega$

Voltage "dropped" between good battery and starting engine is

$$V = IR = 10^2 \cdot 10^{-4} = 10^{-2} \text{ volts. Fine.}$$

$\Rightarrow \Delta V.$

If we used 1mm radius (2mm diameter) cables, then

$$R = \frac{\rho L}{A} = 1.7 \times 10^{-8} \frac{2m}{\pi (10^{-3})^2} = 10^{-8} \cdot 10^6 = 10^{-2} \Omega$$

$$\Rightarrow V = 10^2 \cdot 10^{-2} = 1 \text{ volt } \Rightarrow \text{ only 11 volts at starter. Not enough to work.}$$
The jumper cable example motivates the next topic: electromotive force.

Suppose you try to start your car but find that the battery is "dead." Often you'll see that the electrical parts of the car still work fine, like the radio or headlights. But when you turn the key, the lights dim out while the engine moans.

You might think that if the lights work, the battery must not be dead. In fact, if you measured the voltage at the battery with a voltmeter, you'd probably measure 12 V.

But it is dead, nonetheless. Why... let's study how a battery works.

\[
\begin{array}{c}
  + \quad \frac{1}{+} \\
  - \quad \frac{1}{-}
\end{array}
\]

A battery is drawn like this as a caricature of the cells inside it. It consists of dissimilar metal plates with an acid that uses chemical potential energy to pump charge across.

The key is that it pumps charge from - to + to create a potential.
This pump of charge is called
Emototive force
and written \( E \) or just electromotive force, or EMF.

The name isn't important, and in fact it is a bad name, because it is not a force but a potential and charges move not just electrons. Anyway, it has a potential defined by the chemistry and geometry. 12V in a typical lead-acid car battery comes from 2V per cell based on the chemistry times 6 cells.

But, the current that the battery can produce is limited, e.g. by how many ions can flow through the acid. So a battery is really:

\[
\begin{align*}
\frac{I}{V} & \quad \text{Internal resistance, } \frac{I}{V} \\
- & \quad \text{EMF}
\end{align*}
\]

The symbol for a resistor is also a caricature, long path packed in a small area. (Actually look like this in silicon lithography for large resistance poly-silicon resistors).
Now we can see why a car battery can give 12 V but not start the car, its internal resistance has increased (acid ions depleted).

If \( r_{\text{int}} = 0.1 \Omega \) then

\[
\begin{align*}
0.1 \Omega & \quad \text{stater motor} = 0.1 \Omega \\
12 \text{V} & \quad \text{V} \text{ is } \sim 11 \text{V}
\end{align*}
\]

\[ V = IR \quad \Rightarrow \quad 6 \text{V} \text{ dropped across the } r_{\text{int}} \]
and 6V across the starter, \( \Rightarrow \) insufficient.

But the lights are lower power \( \Rightarrow \) higher R, say 1Ω.

\[ \begin{align*}
\text{V is } \sim 1 \text{V}. \\
1 \Omega & \quad \text{V is } \sim 11 \text{V}
\end{align*} \]

To start the car we need to regenerate the acid. Pushing current backwards reverses the chemical process to regenerate acid.

\[
\begin{align*}
Pb & | \quad SO_4 \quad | \quad PbO_2 \quad \Rightarrow \quad PbSO_4 + e^- \\
PbO_2 + H^+ + SO_4 + e^- & \Rightarrow PbSO_4 + H_2O
\end{align*}
\]
DEMO current in water.

So, how can we measure all this?

First, how could you measure the EMF of a battery?

A "voltmeter" senses the voltage without drawing much current. Ideally it has infinite resistance, so zero current. Real DVM's are GΩ's thanks to op-amps with FET inputs.

So,

\[ I = 0 \Rightarrow V \text{ dropped across } R_{\text{int}} = 0 \]

and we measure EMF.

How could we measure \( R_{\text{int}} \)?

Short across with \( R=0\,\Omega \) and measure \( I \).

\[ I = \frac{EMF}{R_{\text{int}}} \Rightarrow R_{\text{int}} = \frac{EMF}{I} \]

But, don't do this!! or BOOM!!
Instead, we should put a safe, non-zero, resistance in there.

\[ I \text{ is the same everywhere in the circuit, so the } I \text{ that the ammeter measures also flows through } i_{\text{int}} \text{ and } R, \text{ meaning} \]

\[ EMF = I \cdot i_{\text{int}} + IR = I(i_{\text{int}} + R) \]

\[ i_{\text{int}} = \frac{EMF}{I} - R \]

Where we choose } R \text{ so we know its value, and we can find } EMF \text{ as above.}

Actually, we could find both } EMF \text{ and } i_{\text{int}} \text{ using only a voltmeter, not an ammeter.}

\[ i_{\text{int}} = \frac{EMF}{V/R} - R = (\frac{EMF}{V} - 1)R \]
Sample problem:

How much current flows in a 100 W bulb?

\[ P = 100 \, \text{W} = IV = I(110) \]

\[ \Rightarrow I = 1 \, \text{amp}. \]

What is its resistance?

\[ 100 \, \text{W} = I^2R \Rightarrow R = 100 \, \Omega \]

or

\[ 100 \, \text{W} = \frac{V^2}{R} = \frac{(110)^2}{R} \Rightarrow R = 100 \, \Omega. \]

Why are power cords thick?

Power dissipated in cord = \( I^2R \)

\[ R = \frac{P}{L} \]

\[ \Rightarrow \text{make this large} \]

If you put a battery to work for a while, like your cell phone, it gets hot, why?

\[ P_{\text{battery}} = I^2R_{\text{int}} \text{dissipated in the battery.} \]