Sound

Sound is a special case of wave motion.

As we saw last time, sound waves are longitudinal pressure waves. Our ears can hear these for some frequencies. We saw a bit about how last time—and will see some more later.

Today I want to describe some of the sound specific wave properties.

First, what is the speed of sound? It is about 344 m/s or 750 MPH (~640 MPH at altitude) in air. But that is not generally true, just like I can't quote a single number for the speed of a wave on a string, since it depends on tension and μ.

Sound waves have a speed that depends on the medium. We just happen to have a rather constant medium. (But $v = 750 \text{ MPH} \rightarrow 640 \text{ MPH at altitude}$)

I won't derive the speed of sound (or more generally the speed of longitudinal waves in a medium), but let's see that it makes sense.
\[ \nu = \frac{\sqrt{\frac{B}{\rho}}}{\text{speed of a longitudinal wave}} \]

Similar to \( \nu = \sqrt{\frac{T}{M}} \leftarrow \text{force that pushes wave along} \)

Larger \( B \Rightarrow \text{smaller compression for a given force} \)

or more force for a given compression.

\[ \frac{F}{A} = -B \frac{\Delta \nu}{\nu} \text{ from long ago when we discussed stress-strain} \]

Since \( B \) is huge for water, \( \nu_{\text{water}} \gg \nu_{\text{air}} \).

Similarly, \( \nu_{\text{steel}} \gg \nu_{\text{air}} \) (why you can hear the engines inside a supersonic plane).

Although it wasn't worth deriving \( \nu \), it is worth seeing how displacement and pressure are related. We have been writing the wave functions as

\[ y(x,t) \]

But now, the displacement is along \( x \). So it is

\[ x(x,t) \].
To avoid confusion between \( x \) and \( x \), let me call the displacement \( s \). So, a sinusoidally varying wave would be

\[
s(x,t) = s_m \cos(kx - \omega t)
\]

Amplitude, \( s_{\text{max}} \)

Just like transverse waves.

So, how does the pressure vary?

The pressure increase, above normal, for the element is

\[
\Delta p = -B \frac{\Delta V}{V}
\]

Just, old fashioned stress and strain.

The volume of the element is \( V = A \Delta x \).

\( \Delta V \) comes from a difference in the displacement, \( s \), at the two sides (faces) of the element.

\[
\Rightarrow \Delta V = A \Delta s
\]
So,
\[ \Delta p = -B \frac{A \Delta s}{A \Delta x} = -B \frac{\Delta s}{\Delta x} \]

up making it really
infinitesimal.

What is \( \frac{\Delta s}{\Delta x} \)?

\[ S(x,t) = S_m \cos(kx-wt) \]

\[ \frac{\Delta s(x,t)}{\Delta x} = -kS_m \sin(kx-wt) \]

⇒ \( \Delta p = BK S_m \sin(kx-wt) = p(x,t) \)

Pressure change is out of phase with \( S(x,t) \)
and larger by \( BK \).

⇒ Potentially huge

\[ B = 1.4 \times 10^5 \text{Pa for air} \]
\[ B = 2 \times 10^9 \text{Pa for water} \]

Can we write \( BK \) in more useful units?

Recall that \( v = \sqrt{\frac{B}{\rho}} \), so \( BK = v^2 \rho K \).

And \( k = \frac{\omega}{v} \), so

\[ BK = v^2 \rho K = v \rho \omega \text{ or } = \frac{B \omega}{v} = \omega \sqrt{B \rho} \]

Small displacement variation but large pressure
variations. E.g. maximum volume you can stand
is about 28 Pa (\( \approx 10^{-9} \text{atm} \)). This corresponds to
a max displacement of \( \approx 10 \mu m \). Tiny.
I can also express this in terms of $P_{\text{max}}$ instead of $S_{\text{max}}$ (since we saw that $S_{\text{max}}$ is so small -- it is hard to measure).

\[ \rho = \omega \sqrt{\epsilon_0 \sigma} \]

Recall that $P_m = B_k S_m \Rightarrow S_m = \frac{P_m}{B_k} = \frac{P_m}{\omega \sqrt{\epsilon_0}}$

\[ \Rightarrow I_{\text{avg}} = \frac{1}{2} \omega^2 \sqrt{\epsilon_0 \sigma} \frac{P_m^2}{B_k} = \frac{1}{2} \omega^2 \sqrt{\epsilon_0 \sigma} \frac{P_m^2}{\omega^2 B_k} \]

\[ = \frac{1}{2} \frac{P_m}{\sqrt{B_k}} \]

A typical value for $I$ is $I \approx 10^{-5} \frac{W}{m^2}$ for voice.

Minimum detectable $I$, called threshold of hearing, is

\[ I = 10^{-12} \frac{W}{m^2} \equiv I_0. \text{ Very small!} \]

Max bearable is $I \approx 1 \frac{W}{m^2}$.

12 orders of magnitude difference.

Decibels are just a convenient way to keep track of this... convenient once you're familiar.
Define the "sound intensity level" as

$$\beta = 10 \text{dB} \log \frac{I}{I_0}$$

where $I_0 = 10^{-12} \text{ W/m}^2$ is the threshold of hearing.

So if $I = 1 \text{ W/m}^2$, which is the threshold of pain,

$$\beta = 10 \text{dB} \log \frac{1}{10^{-12}} = 10 \text{dB} \log 10^{12} = 10 \text{dB} \times 12$$

$$= 120 \text{dB.} \quad \text{Just 10 times the exponent.}$$

So, what is 20 dB?

$$20 \text{dB} = 10 \text{dB} \log \frac{I}{10^{-12}} \Rightarrow I = 10^{-10} \text{ W/m}^2$$

$$\frac{1}{5}$$

$$50 \text{dB} = 10 \text{dB} \log \frac{I}{10^{-12}} \Rightarrow I = 10^{-7} \text{ W/m}^2$$

Often used in signal processing e.g., amplifying a wireless signal, since you want gains of 10, 100, or 1000, not just 10% or so.
For sounds, the intensity is described with the Decibel scale.

This is a logarithmic scale that makes it more convenient to express enormously different intensities, which is useful since you can hear over an enormous range.

First, though, what exactly does "intensity" mean? Displacement, pressure?

Actually, the intensity of a sound wave is defined by the energy it carries, or more correctly the power per unit area.

\[
\text{Power} = \text{Force} \times \text{Velocity}
\]

\[
I = \frac{\text{Power}}{\text{Area}} = \frac{\text{Force}}{\text{Area}} \times \text{Velocity} = p \nu
\]

\[
P(x,t) = BkS_m \sin(kx - \omega t) = \omega \sqrt{Bf'}S_m \sin(kx - \omega t)
\]

\[
V(x,t) = \frac{\partial S}{\partial t} = + \omega S_m \sin(kx - \omega t)
\]

\[
\Rightarrow I = \omega^2 \sqrt{Bf'}S_m^2 \sin^2(kx - \omega t)
\]

Or \[ I_{\text{avg}} = \frac{1}{2} \omega^2 \sqrt{Bf'}S_m^2 \] (Assumes sinusoidal looks like Power for transverse wave.

**DEMO with ball in vacuum**
Standing waves

We saw standing waves on a string, which arose by adding two traveling waves with just the right phase shift. That is easy to accomplish by reflecting a wave.

Sound waves also reflect so we can similarly make standing sound waves.

DEMO with speaker and pipes.

First closed end.

Wave indicates the displacement

Any # that gives antinode on left & node on the right will resonate.

\[ L = n \frac{\pi}{4} \quad \text{where} \quad n = 1, 3, 5, \ldots \]

or

\[ f = \frac{n v}{4 L} \quad " \quad " \quad " \]

\[ \lambda = \frac{v}{f} \]

\[ L = \frac{n \frac{v}{4}}{f} \]
Next, an open end.
Sound is reflected from the open end just like the closed end—easiest to think of it as an impedance mismatch, or a loose end of a string.

Must have antinode at both ends.

\[ L = \frac{n \lambda}{2} \quad n = 1, 2, 3, \ldots \]

\[ f = \frac{n v}{2L} \quad n = 1, 2, 3, \ldots \]

These are the “displacement” waves. Pressure waves are 90° out of phase with them. So

< Ends open to atmosphere are nodes for pressure
i.e. \( \Delta P = 0 \Rightarrow P = P_{atm} \)

DEMO with flame tube.

DEMO with air organ.

\[ \text{turbulent air flow causes vibrations} \]
Suppose you were standing at an antinode inside the pipe, what would you hear?

Nothing (well nothing from this source at least).

The sound interferes destructively.

If you were at an antinode, it interferes constructively and is louder.

You won't fit in the tube, but we can arrange interference other ways. For example:

Sound waves from A & B add, but B is out of phase and cancels \( \Rightarrow \) NO SOUND. This is like noise cancelling speakers.
So how far apart would I have to place them?

\[ L = \frac{\lambda}{2} = \frac{v/f}{2} = \frac{340 \text{m/s}}{1000 \text{Hz}} \]

\[ \frac{0.34 \text{m}}{2} \approx 17 \text{ cm} \quad \text{Not large.} \]

(Wavelength at 1 kHz \(\approx \frac{1}{3} \text{ meter} \))

The problem is that normal sound is a mix of frequencies. So in reality, just a few frequencies get destroyed, so not too noticeable.

There are, however, some conceptually useful problems e.g., on the homework. Such as

Try with speakers.
Beat frequency

Suppose I add two sound waves of slightly different frequency.

\[ s(x,t) = A \cos(k_1x - \omega_1t) + A \cos(k_2x - \omega_2t) \]

\[ = 2A \cos\left(\frac{k_1+k_2}{2} x - \frac{\omega_1+\omega_2}{2} t\right) \cos\left(\frac{k_1-k_2}{2} x - \frac{\omega_1-\omega_2}{2} t\right) \]

\[ \text{[since } \cos a + \cos b = 2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}, \text{ by trig identity \#2374]} \]

For a given value of \( x \), say \( x=0 \) for simplicity,

\[ s(x=0,t) = 2A \cos\left(\frac{\omega_1+\omega_2}{2} t\right) \cos\left(\frac{\omega_1-\omega_2}{2} t\right) \]

\[ \text{(Remember } \cos(-\theta) = \cos \theta \text{)} \]

\[ \Rightarrow \text{ average freq} \]

\[ \Rightarrow \text{ very low freq.} \]

\[ \Rightarrow \text{ sounds like it is "beating".} \]

Demo with sine wave generator \& guitar.