Physics 2
Review

General Overview -- Applications of Newton's Laws

Rotational Dynamics
\( \vec{\omega}, I, \vec{z}, \vec{L} \) -- know their definition, meaning, use

Statics: Equilibrium Problems
Elasticity: Concepts most important

Gravity: Newton's G law
Kepler's laws
Orbit Calculations
Energy in orbits
Escape Velocity
Force due to extended objects.

Periodic Motion: SHO -- form, examples, \( \omega \rightarrow T \rightarrow F \)
Damped & Forced: Resonance

Fluid Mechanics: Density, Pressure, Buoyancy
Bernoulli's Principle.

Thermodynamics: Temperature, Heat Capacity, Heat Conduction
Convection, Radiation, Thermal Expansion, Phase Change
Molecular model, Eqs of state, Molecular Speed
1st Law: work done, Heat Engines, Refrigerators,
2nd Law, Entropy.
General Comments:

Last quarter we talked about the real fundamentals.

Position — Vectors
Inertia
Force
Momentum
Work & Energy

In general, how things move.

This quarter we extended it to include some

equalities — Not point particles.

Things Stretch

We talked about the first why (gravity) as opposed to how.

Then we described more complicated motion (SITG) and systems (fluids and gases).
Rotational Dynamics

\[ \omega = \frac{d\theta}{dt} \quad \omega = \frac{v}{r} \]

\[ \alpha = \frac{d^2\theta}{dt^2} \quad \alpha = \frac{a \tan}{r} \]

Vectors. Point along rotation axis. Right hand rule.

Kinetic energy of all the moving components, add

\[ K = \sum \frac{1}{2} m_i v_i^2 = \frac{1}{2} \left[ \sum m_i \cdot r_i^2 \right] \omega^2 = \frac{1}{2} I \omega^2 \]

\[ I = \text{I don't memorize formula but know how to combine} \]

Force causes linear acceleration.

What causes rotational (angular) acceleration?

It depends on where the force is applied hence torque.

\[ \vec{\tau} = \vec{r} \times \vec{F} \quad |\vec{\tau}| = |\vec{r}| |\vec{F}| \sin \theta \]

\[ \vec{F} \]

Know this right hand rule stuff.

Can bring 1 right hand to the final!
Newton's 2nd Law gives

\[ \ddot{\mathbf{r}} = \mathbf{F} \]

so we can calculate angular acceleration from torques.

But really, \( \mathbf{F} = m \ddot{\mathbf{r}} \rightarrow \ddot{\mathbf{r}} = \frac{\partial \mathbf{P}}{\partial t} \)

So, \( \ddot{\mathbf{r}} = \mathbf{I} \ddot{\mathbf{\omega}} \rightarrow \ddot{\mathbf{r}} = \frac{\partial \mathbf{L}}{\partial t} \)

where \( \mathbf{L} = \text{angular momentum} = \mathbf{r} \times \mathbf{p} \)

\[ \mathbf{L} = \mathbf{I} \mathbf{\omega} \] for a rigid body.

Just like linear momentum is conserved, angular momentum is conserved.

\[ \frac{d\mathbf{L}}{dt} = 0 \] for a closed system

\[ \mathbf{L} = \mathbf{I} \mathbf{\omega} \]. Change \( \mathbf{I} \rightarrow \mathbf{I}_0 \) changes.

Impulse: \( \int \mathbf{F} \, dt = \Delta \mathbf{p} \)

\( \Rightarrow \int \ddot{\mathbf{r}} \, dt = \Delta \mathbf{L} \)
When we "combine" forces and torques, we get $\ddot{a}$ and $\ddot{\tau}$. (Note "combine" just means when we have forces act on real, extended bodies).

Can solve some problems like $\ddot{a}$ because it is related to $\ddot{\tau}$.

We also saw many problems which dealt with "equilibrium".

$$\sum F = 0 \quad \text{and} \quad \sum \tau = 0$$ about any and all axes.

So given that it is in equilibrium you can determine relation between forces.

Know how to calculate these.

Classic problem solving material.
Then we dealt with elasticity... things are not actually perfectly rigid.

This effect can be approximately modelled by

\[
\frac{\text{stress}}{\text{strain}} = \text{some constant}
\]

if the strain is not too big.

Stress is a force actually \( E/A \).

Strain is a stretch actually \( \Delta L/L \).

\[
\frac{F}{A} = \frac{1}{E} \frac{\Delta L}{L}
\]

\( L \) is a big number for steel ~ 10^4.

Then we talked about Bulk Modulus & Shear Modulus.
Gravity. The first WHY force.

\[ \vec{F} = -G \frac{m_1 m_2}{r^2} \]

\[ G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \]

We derived \( g \) from this in terms of mass and radius of earth, \( \frac{GMm}{R^2} = mg \Rightarrow \frac{GMe}{R^2} = \frac{GM}{R_e^2} \)

We found orbital speeds -- gravity provides centripetal force for constant uniform motion.

Talked about Kepler's empirical laws & how they could be derived.

- Ellipses (not just circles)
- Equal area in equal time
- \( T^2 \propto r^3 \)

Talked about potential & kinetic energy in an orbit and how it allows calculating escape velocity and orbital changes.

\[ \Delta U = -\int F(\vec{r}) \, dr = \int \frac{GMm}{r^2} \, dr = \frac{GMm - GMm}{r_2 - r_1} \]

\[ r_2 = \infty \]
We talked about calculating gravitational force from an extended object.

\[ F = \sum_i \frac{Gm_i m}{r_i^2} \Rightarrow dF = \frac{-Gm_1m_2}{r^2} \]

You should know how to calculate this for simple cases:

- Use \( V \) instead of \( \varepsilon \).

Calculate Lagrange points.

\( \Rightarrow \) Get period = 1 yr.
Periodic Motion

It is common to have \( F(x) = -kx \)

\[ m \frac{d^2x}{dt^2} = -kx \]

\[ \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \]

Has solution \( x(t) = A \cos(\omega t + \phi) \)

Where \( \omega = \sqrt{\frac{k}{m}} \)

There are many examples of this.

- spring
- pendulum
- physical pendulum
- object falling through earth
- floating object
- piston

All are forces \( \Rightarrow ma = \text{constant} x \)

\[ \Rightarrow \omega = \sqrt{\frac{F_{\text{net}}}{m}} \]
If energy is lost to friction, oscillations are damped.

If a periodic force is applied, then the oscillations are driven.

If driving freq matches natural freq, you get a resonance. $\Rightarrow$ increasing amplitude.
Fluid Mechanics

Fluid = liquid, weak cohesive forces

Pressure = $F / A$

actually $P = \frac{dF}{dA}$ if it varies.

Derived: Pressure as a function of depth in a fluid.

\[
\sum F = 0 = PA - (p + \Delta p) A - \rho A \Delta y g
\]

$\Rightarrow \Delta p = -\rho g \Delta y$

Integrate

$\Rightarrow P - P_0 = \rho g h$

Good problem.

And can show that pressure is independent of $x$.

$\Rightarrow$ Pascal's law

\[
P_1 = P_2
\]

\[
\frac{F_1}{A_1} = \frac{F_2}{A_2}
\]

\[
F_2 = \frac{A_2}{A_1} F_1
\]
Derived buoyancy.

\[
\Delta P = \rho gh = \frac{m \rho g}{Ah} = \frac{m_0 g}{A}
\]

\[
= \frac{E}{A} \implies F = \text{Buoyancy} = m_0 g.
\]

Then we added motion to the picture.
Laminar flow of an ideal, incompressible fluid.

Mass in = Mass out \implies \int P_{A_1} v_1 \, dt = \int P_{A_2} v_2 \, dt

\implies A_1 v_1 = A_2 v_2.

Or more generally, include pressure, velocity and height, and we get Bernoulli's equation

\[ P + \rho g y + \frac{1}{2} \rho v^2 = \text{constant}. \]

This is just conservation of energy.
Thermo dynamics

Defined temperature.

Empirical formulas for heat capacity: \( Q = mC\Delta T \)

Heat Conduction: \( \frac{dQ}{dt} = k \frac{\Delta T}{L} \)

Heat of fusion

Heat of vaporization.

Radiation: \( \frac{dQ}{dt} = e \sigma A T^4 \)

Should be able to think about heat transfer mechanisms.

Talked about thermal expansion: \( \frac{\Delta L}{L} = \alpha \Delta T \)

\( \frac{\Delta V}{V} = 3 \alpha \Delta T \)

Then we talked about state equations: \( P, V, T, N \).

For an ideal gas: \( PV = nRT \)

Derived this from a microscopic viewpoint by treating the gas as a large number of non-interacting marbles that bounce off the walls. You should know this derivation.

\[ \Rightarrow \frac{1}{2} m \overline{v^2} = \frac{3}{2} kT \]

\[ \Rightarrow \overline{v_{rms}} = \sqrt{\frac{3kT}{m}} \Rightarrow \text{molecular speeds} \]

\[ = \sqrt{\frac{3RT}{M}} \]
Work and Heat in thermodynamic processes.

\[ W = \int P \, dV \quad \text{just from } F \, dx \]

\[ \Delta U = Q - W \quad \text{Special cases.} \]

Otto cycle (gas engine).
- Isotherm
- Adiabatic
- IsoVol
- Cyclic

Calculate --- derive --- efficiency
\[ \eta = \frac{W}{Q_H} = 1 + \frac{\Delta c_p}{c_p} \]

Carnot cycle, why is it special, what is if, \( \eta = ? \)
Entropy $\Delta S \geq 0$

- # states
- $\int \frac{dv}{T}$ for reversible processes.