Physics 2
Heat & the 1st Law of Thermodynamics

Now that we have an understanding of temperature, heat capacity, and heat flow, we want to apply this to changes in gases.

We saw that \(dq \propto dT\) but not always. E.g. melting and boiling have heat transfer w/o \(dT\).
To simplify things, we'll consider an "ideal gas."

- Internal energy is just kinetic \(\Rightarrow\) temperature.
  (Most important here)
- Doesn't interact with itself

The state of such a gas depends only on its pressure, volume, temperature.

This relation is called the state equation. For an ideal gas it is probably familiar from chemistry.

\[ PV = nRT \]

- \(P\) constant (Rydberg's) \( R = 8.314 \, \text{J} / \text{mol} \cdot \text{K} \)
- # of atoms in gas in moles
  \[ 1 \, \text{mole} = 6.02 \times 10^{23} \, \text{atoms} = N_A \]
  Avogadro's number.\]
we derived this last week, and found that it shows us directly how temperature is internal kinetic energy.

Now, we’ll just use it.

Our goal is to understand heat and energy in a gas.

So, imagine a gas in a piston...

Calculating all the forces inside would be impossible, but we can ignore them and think only about energy, work and heat. That is surprisingly easy.
What work does the gas do?

\[ dW = F \, dy \]
\[ = P \, dV \]
\[ = P \, dv \]

\( dW \) is positive, \( F - dy \) is positive.

\[ \Rightarrow W = \int_{V_i}^{V_f} P \, dV \]

In general, \( P \) is a function of \( V \), through the equation of state: \( PV = nRT \) or \( P = \frac{nRT}{V} \) for an ideal gas.

So, \( P \) is not constant, but is some function of \( V \).

Work is area under curve.

But, work done (area) depends on the path taken.

(Recall conservative v.s. non-conservative forces.)

\[ W = P_i (V_f - V_i) \]

Depends on path \( \Rightarrow \) non-conservative.
In addition to the work done by the gas, we can think about a change in heat transferred to the gas. Like work, heat change depends on the path. E.g.

Let the gas expand from $V_1$ to $V_2$ but keep it at constant temperature as it expands.

Heat must flow into the system. Imagine a small $dW = -dT$ for each $dQ = +dT$.

OR

Gas expands without pushing anything

"Free Expansion"

Find no temperature change in free expansion.

So no heat flow (if insulated)

Same initial & final states, different heat flow.

$\Rightarrow$ Heat transferred to the system, $Q_f$, also depends on the path taken between states $(p_i, v_i)$ to $(p_f, v_f)$.

But, the two are related.

Recall that when we talked about conservation of energy we dealt with non-conservative forces as well and said that this changes the internal energy of the system. Now, we can re-interpret that:

$\Rightarrow$ Heat flow into gas

$\Rightarrow$ Work done by the gas

$\Delta U = Q - W = \text{change in internal energy}$.

And we find that $\Delta U$ is path independent -- just specified by initial and final states.
\[ \Delta U = Q - W \] is the first law of thermodynamics.

What was the zeroth law? \( \Delta T = 0 \) is thermal equilibrium.

We know how to calculate heat and work so we have a macroscopic description of how internal energy changes.

\( U \) (internal energy) is kinetic & potential energy of molecules, but don't need to know that, just heat & work. \( \Rightarrow \) **Change** in internal energy.

Two especially useful cases.

1). Isolated system \( \Rightarrow \) energy conserved \( \Rightarrow \Delta U = 0 \)
   \[ \Rightarrow Q = W \]

2). Cyclic Process. Return to initial state \((p, V)\)
   so \( \Delta U = 0 \) \( \Rightarrow Q = W \).

The cyclic process is important e.g. in an internal combustion engine. We'll discuss this more later.
So, the first law allows us to analyze energy flow in thermodynamic processes.

What is a thermodynamic process?

A change from one state \((P, V, T)\) to another.

\[
P = \frac{nRT}{V}
\]

As usual, we'll consider some simple cases to push the phenomena into a "well lit corner."

In "moving around PV space," I can imagine simple cases.

\[\text{Constant Volume: Called isochoric (also called constant volume).}\]

What is special then? \(W = 0\) since \(dW = PdV\)

\[
\Rightarrow \Delta U = Q = nC_v dT
\]

...same idea as \(mC_dT\)
Constant Pressure: "Isobaric"

What is simplified here? $\Delta V$, $Q$, $W \neq 0$.

$$W = \int P \, dV$$

$P$ constant is an easy integral.

$$= p \Delta V$$

Constant Temperature: Isothermal

Simplification? Not great, but in an ideal gas, $P = \frac{nRT}{V} \implies W = \int P \, dV = \int nRT \frac{dV}{V}$

$$= nRT \ln \frac{V_2}{V_1}$$

Fairly easy.

Also, in an ideal gas, $\Delta U = 0$ if $\Delta T = 0$ (kinetic only).

So, $Q = W = nRT \ln \frac{V_2}{V_1}$
Adiabatic Processes $\equiv$ No change in heat
$\Delta Q = 0$.

Probably the most useful.

How is this realistic?

Fast $\Rightarrow$ No time for heat to flow out of the system.

E.g. Compression of a gas -- heats up but dissipation slow.

$Q = 0$
$\Rightarrow \Delta U = -W$  Energy in or out.

Compress gas $\Rightarrow W < 0$  $\Rightarrow \Delta U > 0$.

Expand gas $\Rightarrow W > 0$  $\Rightarrow \Delta U < 0$.

gas loses energy & does work.
Example: A gasoline engine.

\[ P_1 V_1 T_1 \rightarrow P_2 V_2 T_2 \]

Adiabatic

Burn fuel \( \Rightarrow Q > 0 \) But constant volume.
(Energy was already hidden in the gas, but we can think of it as being added)

Power stroke. Adiabatic
Work done here.

Exhaust & Intake \( \rightarrow \) constant pressure.
Things of course get interesting when you add heat and convert it to mechanical work. Discuss this next time.

A final very special case is a "free expansion". We saw this with the rupturing membrane.

\[ \text{Insulate} \implies q = 0 \implies \text{Adiabatic.} \]
\[ \text{No work done because walls don't move.} \implies w = 0. \]
\[ \therefore \Delta u = q - w = 0. \]

As easy as it gets.

And for an ideal gas; \( U \) depends only on \( T \) (e.g. no rotational energy), so isothermal.
We can revisit heat capacity in an ideal gas a bit:

\[ dQ = nC \, dT. \]

But, \( C \) depends on \( P \div V \). We considered \( C_v \) (constant volume) before. Now we can consider \( C_p \) as well. (How about \( C_T \)?)

First, at constant volume,

\[ dQ = nC_v \, dT \]

\[ \Rightarrow dU = nC_v \, dT \quad \text{because constant volume } \Rightarrow W = 0. \]

\[ 
\Delta U = Q - W 
\]

What if we made it isobaric -- constant pressure?

\[ dQ = nC_p \, dT \]

\[ dQ = ? \quad \text{well, } \quad dU = dQ - dW \]

\[ \Rightarrow dQ = dU + dW \]

\[ nC_p \, dT = dU + dW = dU + pdV \]

\( l \Rightarrow \text{const.} \)

\[ pdV = ? \quad \text{well, } \quad pV = nRT \div p \text{ const.} \]

so \( pdV = nR \, dT \)

\[ \Rightarrow nC_p \, dT = dU + nR \, dT \]

What is \( dU \)? Ideal gas, \( U \) depends only on \( T \) not \( P \) or \( V \).
So, $\text{d}U = nC_v \text{d}T$ same as const volume.

Only diff is in const vol case $\text{d}Q = \text{d}U$ because $\text{d}W = 0$

$\Rightarrow nC_p \text{d}T = nC_v \text{d}T + nR \text{d}T$

$\Rightarrow C_p = C_v + R = \frac{3}{2}R + R = \frac{5}{2}R$

If you let the volume increase, adding heat does work so higher $C_p$ than $C_v$.

Matches measured values pretty well.

It is slightly different for a non-ideal gas, but it is useful to define $C_p$ & $C_v$ for non-ideal gases as well because it is easy to measure them.
In a similar way, we can play with the equations for an adiabatic expansion or compression of an ideal gas.

**Ideal** \( \Rightarrow dU = nC_v dT \) regardless of anything.

**Adiabatic expansion** \( \Rightarrow ? \) \( dQ = 0 \)

\[
dU = -W = -p dV = -\frac{nRT dV}{V}
\]

\[
nC_v dT = -\frac{nRT}{V} dV
\]

\[
\frac{dT}{T} = -\frac{R}{C_V} \frac{dV}{V} \quad \text{but, } R = C_p - C_v
\]

\[
= -\frac{C_p - C_v}{C_V} \frac{dV}{V}
\]

\[
S \Rightarrow \ln T + \text{constant} = -\frac{C_p - C_V}{C_V} \ln V + \text{const.}
\]

\[
\ln T + \frac{C_p - C_v}{C_V} \ln V = \text{const.}
\]

\[
\ln T + \left(\frac{C_p}{C_V} - 1\right) \ln V = \text{const}
\]
Call \( \frac{C_p}{c_v} = \gamma \).

\[ \ln T + (\gamma - 1) \ln V = \text{const.} \]

\[ \ln T + \ln V^{\gamma - 1} = \text{const.} \]

\[ \ln (TV^{\gamma - 1}) = \text{const.} \]

\[ TV^{\gamma - 1} = \text{const.} \]

\[ \frac{pV}{nR} \]

\[ pV^\gamma = \text{const.} \quad \text{For an adiabatic expansion.} \]

Handy. Also wanted to show the power of \( \gamma = \text{constant} \).
Sample Problem

Adiabatic Expansion of an ideal gas.

~ Exhaust of an air hose (or even tail pipe or jet engine).

Why do you see frost or condensation?

High P, small V → small P, high V.

\[ P \rightarrow \text{lower } T. \]

Or \( TV^{\gamma-1} = \text{const.} \)

\[ \frac{T_1 V_1^{\gamma-1}}{T_2 V_2^{\gamma-1}} = \frac{\gamma}{\gamma - 1} \]

\[ \frac{T_2}{T_1} = \left( \frac{V_1}{V_2} \right)^{\gamma-1} \]

\[ \frac{V_2}{V_1} = \sqrt{10^{-4}} \approx 0.01 \]

Note that this is in Kelvin, so \( \approx 2000 \text{ K becomes } 200 \text{ K}. \)
Sample Problem:

Compressed air. You stand on an inner tube.

\[ \begin{align*} \text{sags by 10 cm.} \\
\text{20 cm} \rightarrow \text{10 cm.} \end{align*} \]

How hot does the gas get?

\[ \Delta U = Q - W \rightarrow -mg \Delta h \]

\[ \rightarrow nC_v \Delta T \text{ for an ideal gas.} \]

\[ \rightarrow 0 \text{ for adiabatic} \]

\[ mg \Delta h = nC_v (T_2 - T_1) \]

\[ T_2 - T_1 = \frac{mg \Delta h}{nC_v} \approx \frac{100 \cdot 10 \cdot 0.01}{\frac{1.38 \cdot 8.314}{2}} \approx 10^6 \text{ K} \]
Cloud chamber

Rapid decompression $\Rightarrow$ Colder $\Rightarrow$ Condensation.

Clouds form around small impurities. Find that there are traces.

Cosmic rays.

Mountain Rain.

Lower $p \Rightarrow$ cooler $\Rightarrow$ condense $\Rightarrow$ rain.
Sample problem:

Three isothermal processes are shown for the same, ideal gas. They each have the same initial and final volumes, $V_i$ to $V_f$, but different temperatures.

a) Rank the processes according to the work done by the gas.

b) Rank by change in internal energy.

c) Rank by heat transferred to gas.

\[ \text{a)} \text{ Work} = \text{area under PV curve.} \]

\[ \text{b)} \text{ Isothermal} \Rightarrow \Delta U = 0 \]

\[ \text{c)} \Delta U = 0 \Rightarrow Q = W. \]

\[ dW = pdV = nRT \frac{dV}{V} \]