Fluid Mechanics

Intro
Density
Pressure
Buoyancy
Surface Tension
Bernoulli's Principle

Three states of matter: solid, liquid, gas.

Solid = definite volume and shape
Liquid = definite volume but conforms to container
Gas = volume & shape of container.

There are in fact materials that don't fit this simple classification.
Glass -- old cathedral glass has clearly flowed.

Today we'll discuss the behavior of a more general class of materials which are fluid.

A fluid is a collection of molecules that are randomly arranged, held together by relatively weak cohesive forces and the walls of a container.
We'll find that we can describe quite a bit about the behavior of fluids just using what we already know about force and energy. We'll derive some relations for the behavior of fluids which are just applications of Newton's laws.

First some definitions.

Density = mass/volume. \( \rho = \text{"rho"} \frac{\text{kg}}{\text{m}^3} \)

Many fluids are nearly incompressible so density is a constant. Not always, of course.

Water \( \approx 1.00 \times 10^3 \text{ kg/m}^3 \) at 0°C

Less dense when warmer, but only slightly.

Also only slightly compressible.

Gases like air are very compressible.

Density is, of course, a property of all matter not just fluids, but it is useful in our discussion of fluids, so worth a bit of discussion.

\[
\rho_{\text{water}} = 1.00 \times 10^3 \frac{\text{kg}}{\text{m}^3} = \frac{? \text{ g}}{\text{cm}^3}
\]

\[
= 1.00 \times 10^3 \frac{\text{kg}}{\text{m}^3} \times \frac{1000 \text{ g}}{1 \text{ Kg}} \times \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^3
\]

\[
= 1.00 \times 10^3 \times 10^3 \times 10^{-6} = 1.00 \text{ g/cm}^3
\]
What is more dense than water?

Metals - $\text{Al} = 2.7 \times 10^3$, $\text{Iron} = 7.86 \times 10^3$
$\text{Lead} = 11.3 \times 10^3$, $\text{Gold} = 19.3 \times 10^3$

$\text{Sea Water} = 1.03 \times 10^3$

Less dense?

$\text{Ice} = 0.917 \times 10^3$
$\text{Air} = 1.29$, $\text{Helium} = 0.179$, $\text{Hydrogen} = 0.09$

The density of substances is often given as relative to water, called "specific gravity," even though that is not a very good name.
Pressure. Fluids exert pressure, important concept in fluid mechanics.

\[ P = \frac{F}{A} \]

but, only forces perpendicular to the area matter, otherwise no action. Normal force

So,

\[ P = \frac{F}{A} \]

And of course, the force may not be the same everywhere.

So,

\[ P = \frac{\Delta F}{\Delta A} \]

Force is a vector, but pressure is not. It always is perpendicular to surface.

The units are \( \text{Force} = \frac{N}{m^2} = \text{Pascal, Pa} \) or \( \text{Pressure} = \frac{N}{m^2} = \text{Pascal, Pa} \). Just as we saw when dealing with stress and strain.
As you know from popping ears in a pool or airplane, pressure in a fluid increases with depth. Let's see why.

Bottom surface is at $y$, Top is at $y+dy$.

Weight of the fluid element is $dmg = \rho A dy g$

Pressure on bottom surface, we'll call $p$.
Call the pressure on the top surface $p + dp$.

The question of course is what is $dp$, 0?

Well, if the fluid is in equilibrium, then

$$\sum F = 0 \Rightarrow \sum F_y = 0 = \text{All forces acting on element.}$$

$$0 = pA - \frac{(p + dp)A}{\gamma} - \rho A dy g$$

Force up on bottom surface
Force down on top surface
Force of gravity

$$\Rightarrow \ dp = -\rho g \ dy$$

$$\frac{dp}{dy} = -\rho g$$

or $p_2 - p_1 = -\rho g (y_2 - y_1)$

or $p = p_0 + \rho gh$
Pressure increases linearly with depth in a uniform fluid.

If the density changed with depth then it would be a bit more complicated, although same procedure. (Only changes slightly).

Can also see that pressure is independent of x. Same pressure at all points with the same height.

Useful. Called Pascal’s law. (Though not a law really, just a restatement of Newton’s laws)

\[ F_2 = \frac{A_2}{A_1} F_1 \]

Small force, \( F_1 \), on a small area \( A_1 \), causes a larger force over a larger area.

Concept behind a hydraulic jack.

Of course need to push farther... energy conserved.
\[ P = P_0 + \rho gh \] is used to measure pressure.

Simpliest example is a manometer (pressure meter).

\[ P = P_0 + \rho gh \]

If \( P_0 = 0 \) (vacuum) then get a direct measure of \( P \). That is how atmospheric pressure is measured. Usually quoted as "inches of mercury" \( \approx 30 \) inches.

\[
1 \text{ atm} = 1.01 \times 10^5 \text{ Pa} = 1.36 \times 10^3 \text{ kg/m}^3 \times 9.8 \frac{\text{m}}{\text{s}^2} \times h
\]

\[ \Rightarrow h = 0.76 \text{ m} \]

What if we used water? \( h \) is about 13.6 times as high.

Atmospheric pressure can support \( \sim 10 \text{ m} \) of water!

And, \( 10 \text{ m} \) under the water in the ocean, the pressure is 2 atm.
Questions:

Is your blood pressure higher in your head or your feet?

Your feet -- head is closer to heart... so there is flow differences, but if only weight...

In a scuba diving adventure, how could you figure out which way is up?

Measure pressure, and go toward lower pressure.

Of course, there is an easier way... follow the bubbles. Because the bubbles are buoyant.

Buoyancy -- things lighter (less dense) than water float.

Archimedes Principle -- fluid exerts an upward force on a submerged body equal to the weight of the water displaced by the body.

Let's derive that.
\[ \Delta P = P_b - P_{top} = \rho g h = \frac{m f}{Ah} gh = \frac{m_f g}{A} \]

\[ \Delta P = \frac{F}{A} \Rightarrow \text{force up on body} \]

\[ B = \text{buoyancy} \]

Of course, the weight of the body doesn't go away, so, the net force is

\[ B - mg = (m_f - m) g \]

\[ = (\rho_f - \rho) V g \]
Conceptual Questions.

Suppose you have a cup of water with ice floating in it. Then you step in an elevator and accelerate upward.

Does the ice float higher, sink, or stay at the same level?

No change in level.

\[ M_{\text{water}} \cdot g = M_{\text{ice}} \cdot g \]

Can use this to survive enormous accelerations, e.g. required to reach "interstellar" velocities.

Suppose a damaged ship has filled with sea water so that it is just barely afloat.

It is towed to dock at a shipyard in a bay that is fed by a fresh water river. What happens when the boat gets into fresh water.

\[ M_{\text{sea water}} \cdot g = M_{\text{boat}} \cdot g \]

\[ \rho_{\text{seawater}} > \rho_{\text{fresh}} \text{ so it sinks.} \]
Note that potential energy is more positive the deeper you go.

For a helium balloon, how high does the balloon float?

This is different from water, because air changes a lot with height.

\[ m_{bg}g = m_{air}g = \rho_{air} V g \]

For a submarine, how do you surface and dive.

Change avg \( P \) to be \( > \) or \( < \) \( P \) water.

by letting in water
or pumping it out
or replacing it with air.
If you are in a boat and then throw an anchor over board so it sinks to the bottom,
- Does your boat go up, down, same?
  Up -- takes less displaced water to balance your reduced weight.
- Does the level of the pond go up, down, same?
  In the boat, the anchor displaces a volume of water = to its weight.
  On the bottom, the muck supports its weight, so it displaces a volume of water = to its volume. < before.
  So, the level decreases.

When you are swimming is there more buoyant force after inhaling or exhaling (air that is)?
  Inhaling -- more volume.

Two identical ships are sailing. One is empty, the other is loaded with styrofoam. Which one floats higher in the water?
  The empty one. The styrofoam is inside air lighter than styrofoam.
A barge loaded with gravel goes down a river. It approaches a bridge and it becomes clear to the captain that the top of the gravel pile is going to hit the bottom of the bridge. What should he do?

- Take on water.
- Or stop at a sand-bar and pile on more gravel.
Force on a dam

\[ p = p_0 + \rho gh = \rho gh \]

So, force on a small region \( dy \) at height \( y \) is

\[ dF = p \cdot dy = \rho g (H - y) \, dy \]

\[ F = \int_0^H \rho g (H - y) \, dy = \rho g \left( H y - \frac{y^2}{2} \right) \bigg|_0^H \]

\[ = \rho g H^2 - \frac{1}{2} \rho g H^2 \]

\[ = \frac{1}{2} \rho g H^2 \]

\[ \Rightarrow \text{quadratic with height.} \]

\[ \Rightarrow \text{linear with width.} \]

That is total force.

Force on a small area at the bottom depends only on \( H \).
Fluid Dynamics

An ideal fluid is incompressible and has no internal friction. As usual, we'll start with the ideal case and then deal with the complications.

Consider the flow of a fluid.

Path of an individual particle is called a flow line.

If there is "steady flow", then any subsequent particle on that path follows the same path. The path is called a "stream-line". (Tangent to stream line is velocity at that point).

Expand this flow line to a flow tube. In steady flow, no fluid can cross the walls of a flow tube... as if a separate pipe.

This is laminar flow

flows in sheets.

Turbulent flow is the term for the more chaotic mixed flow.
Continuity Equation ... flow rate.

\[
\frac{A_1 \cdot v_1 \, dt}{v_2 \cdot dt} = A_2
\]

Mass in = mass out, so if incompressible, volume in = volume out.

\[m_1 = m_2 \quad \Rightarrow \quad \int A_1 v_1 \, dt = \int A_2 v_2 \, dt\]

\[A_1 \cdot v_1 = A_2 \cdot v_2\]

If you squeeze the area, the fluid flows faster.

E.g. hose, river.

\[\frac{dm}{dt} = \rho \cdot A \cdot v\]

\[
\frac{\partial V}{\partial t} = AV \quad \text{volume flow rate}
\]

If it were not incompressible (double negative), then how do we generalize? Still mass in = mass out, so

\[\rho \cdot A_1 \cdot v_1 = \rho_2 \cdot A_2 \cdot v_2\]
**Bernoulli's Principle**

If we apply what we know about energy, work, etc. to a moving fluid, we can get Bernoulli's principle.

Pressure $p_1$ applies a force $p_1 A_1$,

$$F_1 = p_1 A_1$$

and $F_2 = -p_2 A_2$.

Work done by these forces is:

$$W_1 = F_1 d = p_1 A_1 \Delta x_1 = p_1 \Delta V_1$$

$$W_2 = -p_2 A_2 \Delta x_2 = -p_2 \Delta V_2$$

But we just saw that $\Delta V_1 = \Delta V_2$ for an ideal fluid.

Work due to gravity is

$$W = -mg (y_2 - y_1) = - \rho g \Delta V (y_2 - y_1)$$

Net work:

$$W_{net} = P_1 \Delta V - P_2 \Delta V - \rho g \Delta V (y_2 - y_1)$$
What is the net work done equal to?

Work-energy theorem. \( W_{net} = \Delta K \)

\[
P, \Delta V - P_2, \Delta V - \rho g (y_2 - y_1) \Delta V = \frac{1}{2} \Delta m v_2^2 - \frac{1}{2} \Delta m v_1^2
\]

\[
\rho \Delta V
\]

\[
P_1 - P_2 - \rho g (y_2 - y_1) = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2
\]

\[
\Rightarrow P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2
\]

OR

\[
P + \rho g y + \frac{1}{2} \rho v^2 = \text{constant}
\]

This is really just conservation of energy, Newton's laws, \( \frac{1}{2} \) work-energy theorem flowed together.

Not new... but a much more useful form.
Conceptual Question

When you are riding a bike along the road and you get passed by a truck, you first feel pushed away from it, then pulled in. Why?

\[ P_1, V_1 \uparrow \quad \Rightarrow \quad P_2, V_2 \]

\[ P_1 + \frac{1}{2} P V_1^2 = P_2 + \frac{1}{2} P V_2^2 \]

\[ V_1 > V_2 \quad \Rightarrow \quad P_1 < P_2 \quad \text{it really sucks.} \]

Blow beside paper...

Related question: A tornado passes your house... tightly sealed. What happens to the house? It pops. Ex not implode.

high \( V \) = small pressure.

\[ \uparrow \rightarrow \downarrow \leftrightarrow \]
This is ~ the same as on a wing -- it is more complicated but the idea holds.

Higher v $\Rightarrow$ lower P.

Air pump

or fire ex'ting.

Sewer

Disk washer

Static

Pitot tube

$-\Delta h \Rightarrow \Delta P$

Plaque $\Rightarrow$ fluttering flow
Sample Problem

What is \( v \), velocity of ejected fluid?

At surface, \( z \) at spot, Bernoulli's equation is same.

\[
P_2 + \frac{1}{2} \rho v^2 + \rho g y_2 = P_1 + \frac{1}{2} \rho v^2 + \rho g y_1
\]

\[
\frac{1}{2} \rho v^2 = (P_2 - P_1) + \rho g (y_2 - y_1)
\]

\[
v^2 = \frac{2 (P_2 - P_1)}{\rho} + 2gh
\]

\[
v = \sqrt{\frac{2 (P_2 - P_1)}{\rho} + 2gh}
\]

Limits ok? \( P_2 - P_1 = 0 \) \( \Rightarrow \sqrt{2gh} \) ... just energy relation.

\( g = \frac{1}{2} \) \( P_2 < P_1 \) \( \Rightarrow \sqrt{-\frac{2}{\rho} |P_2 - P_1|} \) ?! Just negative \( v \).
Sample Problem:

What happens?

oscillates -- what freq?

\[ B = m_{\text{air}} g - m_{\text{g}} g = (P_{\text{air}} - P_{\text{b}}) V g \]

\[ B_x = -(P_{\text{air}} - P_{\text{b}}) V g \frac{x}{L} \]

\[ \Rightarrow \text{SHM} \quad \text{freq} = \frac{1}{2\pi} \quad \omega = \frac{1}{2\pi} \]

\[ m \frac{d^2x}{dt^2} + (P_{\text{a}} - P_{\text{b}}) V \frac{g}{L} x = 0 \]

\[ \Rightarrow \quad \omega = \sqrt{\frac{(P_{\text{a}} - P_{\text{b}}) V g}{L \cdot m}} \]
Curve Ball - How does it work?

Top view

No Rotation

Air is carried with the ball's rotation (only true due to viscosity of air - can't ignore friction here)

Net result is "movement" of the ball (in this case sideways)

Q: What if this was a sideview? What kind of pitch is that? "Rising Fastball"