Review

The final will cover all of the course. It will involve conceptual questions and problems like the two midterms.

I’ll spend about 30 minutes reviewing everything we’ve covered. My notes will be posted on the web. So, don’t focus on writing this all down. Rather, think about it as we go through and try to predict what is next and how it all fits in a big picture.

Kinematics is the attempt to describe motion. Dynamics is the attempt to describe the cause of motion.

These are the topics we covered. Along the way we saw some fundamental principles like conservation of energy and momentum.

First we described the position of an object. This involved coordinate systems, vectors, and units. Position is described by \( \mathbf{x} \) and \( t \).

Motion is the change of position.

\[
\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t}
\]

It is a vector \( \Rightarrow \) 3 independent components.
Similarly,
\[
\ddot{x}(t) = \frac{d\dot{x}(t)}{dt}
\]

What we really want to do is predict \( \dot{x}(t) \) or \( \ddot{x}(t) \) from knowledge of \( \ddot{x}(t) \), so we integrate instead.
\[
\ddot{x}(t) \rightarrow \dot{x}(t) = \int_0^t \ddot{x}(t') \, dt' \rightarrow x(t) = \int_0^t \dot{x}(t') \, dt'
\]

We talked a lot about motion with constant acceleration because:

1) Mathematically easy but sufficient to see concepts.
2) Good description of motion in free-fall.

\[ a = -g \]

\[
\begin{align*}
Z(t) &= Z_0 + v_{z_0} t + \frac{1}{2} a t^2 \\
y(t) &= y_0 + v_{y_0} t \\
x(t) &= x_0 + v_{x_0} t
\end{align*}
\]

A very important feature is that the motion in \( Z \) does not depend on the motion in \( y \) or \( x \), and vice versa.

We had several questions on this in midterm exams. It is likely to be on the final as well.
Important Concepts

1. Motion in orthogonal directions is independent.

\[ \begin{align*}
&v_x(t) = \frac{dx}{dt} \\
&a_x(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}
\end{align*} \]

For example,

1. If you throw your keys to someone who is being hoisted at constant speed by a crane, draw the height of your keys above ground as a function of time.

- Constant speed means straight \( z(t) \).

- Thrown keys are in free fall while in the air so they follow a parabola in \( z(t) \).
Another example:

Ball thrown up, falls and sticks in the mud.
When is the acceleration zero? When is the velocity zero?

Likely problems are: to calculate time required to follow some path, velocity vs. distance, range.

I said that independence of motion in each direction is a key concept. But, sometimes the motion is constrained to lie in a circle.

Because we can always choose our coordinate system as we wish.
Motion in a circle with uniform speed has
\[ \dot{\mathbf{r}} = R \hat{\mathbf{r}} \]
\[ \mathbf{v} \] perpendicular to \( \mathbf{r} \)
\[ \mathbf{a} \] perpendicular to \( \mathbf{v} \)
\[ \mathbf{a} = \frac{\mathbf{v}^2}{R} \hat{\mathbf{r}} \]

\( \mathbf{a} \) pointed towards center

This is not magic, it is simply a statement (or rather a mathematical consequence) of the fact that the motion remains in a circle.

"In order for something to be moving in a path which is a circle of radius \( R \), its velocity must change with a rate \( \mathbf{a} = \frac{\mathbf{v}^2}{R} \). Faster \( \mathbf{v} \) but same \( R \) requires higher \( |\mathbf{a}| \)."
Force and Newton's Laws

Change in motion is caused by forces.

⇒ With no force things will move at constant velocity forever. That velocity could be zero. In fact, I can always choose a moving coordinate system such that any object with no force on it is not moving in that frame.

"No force" really means \( \sum F = 0 \) and forces are vectors.

\[ \sum F = F_{\text{net}} = ma \]

Forces cause acceleration, not motion, acceleration.

3rd law is that every force has an equal & opposite reaction force. These action-reaction pairs act on different objects.
We worked a lot of problems involving force vectors, free-body diagrams, etc.

You should know:

1) Static Equilibrium Problems such as

(Tension in rope?)

$\sum F = 0$

Since not moving

2) Find Acceleration in cases where we don't know that it is zero.

3) Including friction.
Then we talked about work and energy.

\[ \triangle F = m \Delta \vec{v} \]

and predict

is sufficient to describe the motion of an object for all time if we knew \( \dot{\vec{x}}(t) \).

That works in inertial guidance systems (accounting) but is not satisfyingly predictive, because:

1) We don't usually know \( \dot{\vec{F}}(t) \). Instead we know \( \vec{F}(x) \), or we only know averages.

2) Integrals to get from \( \overrightarrow{\Delta x} = \int \vec{F}/m \) to \( \vec{x}(t) \) can be very difficult.

So, we played with the meaning of \( \int \vec{F}(x) \cdot d\vec{x} \) and \( \int \dot{\vec{F}}(t) \cdot dt \). That lead to very useful ideas & tools.

1) Conservation of Energy

Potential + Kinetic + Internal is constant

\[ \Delta U = \text{Work Done} = -\int \vec{F}(x) \cdot d\vec{x} \]

\[ \text{Kinetic} = \frac{1}{2} m V^2 + \frac{1}{2} I \omega^2 \]

Recently saw that this is a simple way to express kinetic energy of rotational velocity.

Internal = - Work done other than conservative.
There are many problems we can work with this:

Inclines, springs, pulleys, Artificial Forces.

With or without friction

With or without pulley mass (now that we have $k = \frac{1}{2} I \omega^2$)

Note that I said, "Artificial forces." It is important to know that we can solve things this way for any force.

$U = mgh$ is not magic, it is just $U = -W = -\int_{h_0}^{h} (-mg) \, dz$.

We talked about using potential $U(x)$ instead of force.
We can always find the force, $F_x(x) = -\frac{dU}{dx}$, if needed.

But potential is a more convenient viewpoint because conservation of energy makes energy diagrams a powerful way to look at the behavior of a system.
Energy Conservation is a powerful tool for analyzing motion. You should know how to use it to solve problems.

\[ U_i + K_i = U_f + K_f + U_{int} \]

[Diagram of a ramp and projectile motion]

How far from launching point does it land?

Messy, but conceptually simple.

- Find speed at point where it goes into free-fall. From there it is just free-fall.

\[ U_i = mgh \quad \frac{1}{2} K_i = 0 \]
\[ U_f = \frac{1}{2} mgh \quad \frac{1}{2} K_i = \frac{1}{2} m v^2 \quad \frac{1}{2} U_{int} = 0 \text{ if no friction.} \]

\[ \frac{1}{2} m v^2 + \frac{1}{2} mgh = mgh \]

\[ v = \sqrt{gh} \]
How far then?

Well time to fall $h/2$ is

$$z = z_0 + v_{z0} t - \frac{1}{2} g t^2$$

$$\Rightarrow 0 = \frac{h}{2} + 0 - \frac{1}{2} g t^2$$

$$gt^2 = h$$

$$t = \sqrt{\frac{h}{g}}$$

$$x = x_0 + v_{x0} t$$

$$\Rightarrow x = \sqrt{gh} \cdot \sqrt{\frac{h}{g}} = \sqrt{gh^2/g} = h$$

"Spooky" ... but easy.

What if we included friction? Then $V_{int}$ would have $\mu mg$ term. Harder, but still conceptually easy.
Then, we talked about momentum.

\[ \vec{p} = m \vec{v} \]

and

\[ \Delta \vec{F} = m \Delta \vec{a} \] is actually \[ \Delta \vec{F} = \frac{d\vec{p}}{dt} \]

So, analogous to \( W = \int \vec{F}(x) \cdot d\vec{x} \), impulse is

\[ \Delta \vec{p} = \int \vec{F}(t) \, dt \]

This is useful because we derived conservation of momentum, and need only use that rather than trying to determine \( \vec{F}(t) \).

In hard collisions, \( \Delta \vec{p} \) from the short, "hard" forces dominates over any other forces like gravity, so external forces can be neglected.

\[ \vec{p}_i = \vec{p}_f \] (Generalize to four-vector to include \( E \)).

If "elastic", then kinetic energy is also conserved, and we can determine final motion if we know initial, and at least a little about the final (e.g. angles).
So, we could for example ask what happens in this case.

We found ourselves talking about bullets hitting blocks a lot, of course any example would work.

**Ballistic Pendulum** -- momentum transferred from bullet to block is a good sample problem that uses both conservation of momentum and conservation of energy.

\[ \frac{MV}{D} \rightarrow (M+m)gh \rightarrow (M+m)V' \rightarrow \frac{1}{2}(M+m)V'^2 \]
Finally, we talked about rotational kinematics and derived an expression for the kinetic energy in rotational motion.

\[ K_{\text{rot}} = \frac{1}{2} I \omega^2 \]

Which goes into conservation of energy just like linear kinetic energy.

Our double incline example could be extended to be a ball rolling.

\[ E_{\text{initial}} = mgh \quad E_{\text{final}} = \frac{1}{2} mgh + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \]

Where \( I_{\text{sphere}} = \frac{2}{5} M R^2 \) and \( \omega = v/R \)

\[ mgh = \frac{1}{2} mgh + \frac{1}{2} m v^2 + \frac{1}{5} m v^2 \]

\[ v = \sqrt{5gh/7} \]
Next quarter you'll cover:

- Rotational Dynamics
- Deformation of "not so rigid" bodies
- Gravity and planetary motion
- Periodic Motion
- Thermal Properties