Review

Equations of motion

Given \( \ddot{v}(t) \), \( \int \) to find \( \dot{v}(t) \), then \( \int \) to find \( v(t) \).

If constant \( a \), then it is easy. Worked many examples of constant \( a \) due to gravity:

\[
\begin{align*}
Z(t) &= Z_o + V_{z_0} t - \frac{1}{2} g t^2 \\
X(t) &= X_o + V_{x_0} t \\
Y(t) &= Y_o + V_{y_0} t
\end{align*}
\]

Motion in each direction separates.

Freefall from height \( h \) with initial speed 0.

Would hit the ground when,

\[
Z(t) = 0 = h + 0 - \frac{1}{2} g t^2
\]

\[
\Rightarrow t = \sqrt{\frac{2h}{g}}
\]

What if there is an initial speed? If there is any in the vertical direction, we have to include it here. Otherwise... x or y velocity don't matter.
Given $x(t)$, you can find $v(t)$ by taking the derivative. Velocity is just the slope.

When are the velocities equal?

How about

When are the velocities equal?

What can you say about their motion?

$v(t)$

$a(t)$

Circular motion

$\vec{v}$ is always perpendicular to path.

$\vec{a} = -\frac{v^2}{r} \hat{r}$
Forces & Newton's Laws

\[ \Sigma \vec{F} = m \vec{a} \]

\( \Rightarrow 1) \) Without a force, no change in velocity.

\( 2) \) Net force is what matters.

\( 3) \) Vectors

Newton's 3rd Law

Action-reaction... The reaction must be acting on a different object. Weight \( \rightarrow \) pull on earth's normal force... e.g. when floor collapses.

Weight = \( mg \) \quad \text{Your weight} \approx 50 \text{Kg} \cdot 10 \text{ m/s}^2 = 500 \text{ N}

Your mass = 50 kg

Sample Problems

What is \( T_1 \) \& \( T_2 \)?

\[ T_2 \sin \theta = mg \Rightarrow T_2 = \frac{mg}{\sin \theta} \]

\[ T_2 \cos \theta = T_1 \Rightarrow T_1 = \frac{mg}{\tan \theta} \]

What are forces on block, what is its acceleration.

\( \vec{F} = mg \sin \alpha \) along slope.

\( F_x = mg \sin \alpha \cos \alpha \)

\( F_y = mg \sin^2 \alpha \cos (\theta + \alpha) = mg \sin \alpha \sin (-\alpha) = -mg \sin^2 \alpha \)
We talked about friction & drag.

Friction → static or kinetic

\[ |F| = \mu |N| \text{ in direction opposite to motion (relative to surface)} \]

\[ M_s > M_k \]

Drag is an air resistance opposing motion just like friction.

\[ F = \frac{1}{2} D \rho Av^2 \]

As \( v \) increases, this force eventually matches \( mg \) and acceleration stops → terminal velocity.

Force in circular motion

\[ \ddot{a} \text{ points in } \Rightarrow \vec{F} \text{ points in} \]

To maintain constant circular motion we need

\[ \ddot{a} = -\frac{v^2}{r} \hat{r} \]

\[ \text{So,} \quad \vec{F} = -\frac{mv^2}{r} \hat{r} \]

That force comes from, string tension, gravity, normal force, etc.
Work and kinetic energy.
\[ W = \vec{F} \cdot \vec{d} = F d \cos \theta \]

If \( F \) is a function of \( x \), we can integrate it to get the work.
\[ W = \int_{x_i}^{x_f} F(x) \, dx \]

A spring is a good example, \( F(x) = -kx \).
\[ \Rightarrow W_{\text{spring}} = \int_{x_i}^{x_f} -kx \, dx = -\frac{1}{2} k x_f^2 + \frac{1}{2} k x_i^2 \]

Or, we can imagine other \( F(x) \) scenarios, and calculate work done by each force.

Work-Energy Theorem.

The concept of work is useful to solve problems because of the work-energy theorem.
\[ W_{\text{net}} = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2 \]

\( W_{\text{net}} = \) either sum of work done by each force or work done by vector sum of all forces.
Some Problems

How fast is the mass moving when it reaches the end, where the spring is relaxed?

1) Forces on block

\[ F(x) = -kx - mg \sin \alpha - \mu mg \cos \alpha \]

Speed at \( x=0 \)? Use work energy theorem,

\[ \int_{-x_0}^{0} F(x) \, dx = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \quad \text{at rest} \]

\[ \frac{1}{2} m v_f^2 = \int_{-x_0}^{0} -kx - mg \sin \alpha - \mu mg \cos \alpha \, dx \]

\[ = -\frac{1}{2} k x^2 - mg (\sin \alpha + \mu \cos \alpha) x \bigg|_{-x_0}^{0} \]

\[ = 0 + \frac{1}{2} k x_0^2 + mg (\sin \alpha + \mu \cos \alpha) (-x_0) \]

\[ = \frac{1}{2} k x_0^2 - mg x_0 (\sin \alpha + \mu \cos \alpha) \]

\[ v = \sqrt{\frac{k x_0^2}{m} - 2g x_0 (\sin \alpha + \mu \cos \alpha)} \]
Pulley Problem

1) Coordinate system

2) Free body diagrams:
   \[ F = m \alpha \]
   \[
   m_1 \ddot{y}_1 = -m_1 g + T \\
   m_2 \ddot{y}_2 = -m_2 g + T
   \]

3) Constraint
   \[
   L = -y_1 - (-p_1) + \pi R - y_2
   \]
   \[
   \frac{dl}{dt} = 0 = \dot{y}_1 - \dot{y}_2
   \]
   \[
   \Rightarrow \dot{y}_1 = -\dot{y}_2 \quad \text{Make Sense?}
   \]
   \[
   \ddot{y}_1 = -\ddot{y}_2
   \]

4) Solve
   \[
   T = m_1 \ddot{y}_1 + m_1 g
   \]
   \[
   \Rightarrow m_2 \ddot{y}_2 = -m_2 g + m_1 \ddot{y}_1 + m_1 g
   \]
   \[
   (m_1 + m_2) \ddot{y}_2 = g (m_1 - m_2)
   \]
   \[
   \ddot{y}_2 = g \left( \frac{m_1 - m_2}{m_1 + m_2} \right)
   \]
   \[
   \ddot{y}_1 = \frac{m_2 - m_1}{m_1 + m_2}
   \]

Check answer
Bungee Jump

$Z = 0$ where spring is relaxed.

Speed at $Z = 0$?

$V = \sqrt{\frac{100g}{100}} = 3.2 \text{ m/s}$

$\frac{100g}{100} = 31.3\%$

all your stop?

$-mgz - \frac{1}{2}KZ^2 = \frac{1}{2}m(100g)$

$S_{0} F_{x}(Z) dx$

So, find speed at $Z = 0$

Burned

So, $S_{0}$

$-mgZ - \frac{1}{2}KZ^2 = 0$

$50 - \frac{1}{2}gZ^2$

$E = 0$

$g$ gives a work of $-\frac{1}{2}mV^2$.

you stop.

$\frac{1}{2}m(100g)$

$V = \sqrt{\frac{100}{100}} = 3.2 \text{ m/s}$

$\frac{100g}{100} = 31.3\%$

$\text{all your stop.}$

$z = z_{0} + \frac{V_{0}}{g}t - \frac{1}{2}gt^{2}$

When you jump, you stretch it when

$S = 5 \text{ m}$

are you used.

$100Z = 500$

$500 = 500$

$F_{spring} = \frac{100f}{m}$

$= 100 \text{ N/m}$

Spring constant

You mass $= 50 \text{ kg}$

Spring length $= 50 \text{ m}$

+ $50 \text{ m}$

$\text{jump from + 50 m}$

You stretch it when
Sample Problem

Write expression for acceleration of each block.

1). Coord System

2). Free body diagrams to find net force.

3). 2nd law to find acceleration expression.

\[
\begin{align*}
\Rightarrow m_1 \ddot{x}_1 &= T_1 - m_1 g \\
M_2 \ddot{x}_2 &= T_2 - T_1 \\
M_3 \ddot{x}_3 &= T_2 - m_3 g
\end{align*}
\]

5 eqns
5 unknowns
⇒ physics is done

\[\ddot{x}_1 = \ddot{x}_2 = -\ddot{x}_3\]

Angles don't matter. Why not? "Because we didn't need them."

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Other problems:

1) \[ \begin{array}{c} K_2 \\ \hline M \\ \hline M \\ \hline m \\ \hline x_1 \end{array} \]

No friction.
Spring 1 has constant \( K_1 \).
Spring 2 has constant \( K_2 \).
When spring 1 launches the mass, how far will it compress spring 2?

Solution: Use work energy theorem.

Work done by spring 1:
\[ \frac{1}{2} K_1 x_1^2 = \frac{1}{2} m v^2 - 0 \]

By W.E.T.

Work done by spring 2:
\[ W_2 = -\frac{1}{2} K_2 x_2^2 + \frac{1}{2} K_2 (0)^2 = \frac{1}{2} m v_0^2 - \frac{1}{2} m v^2 \]

\[ S_0 - \frac{1}{2} K_2 x_2^2 = - \frac{1}{2} m v^2 = - \frac{1}{2} K_1 x_1^2 \]

\[ x_2 = \sqrt{\frac{K_1}{K_2}} x_1 \]

Check units

Check limits.

How about the case where there is friction?